Redistribution under General Decision Rules[*][†]

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Abstract

We study the political economy of redistribution over a broad class of decision rules. Since the core is generically non-unique, we suggest a simple and elegant procedure to select a robust equilibrium. Our selected policy depends on the full income profile, and in particular, on the preferences of two *decisive* voters. The effect of increasing inequality on redistribution depends on the decision rule and the shape of the income distribution; redistribution will increase if both decisive voters are 'relatively poor', and decrease if at least one is sufficiently 'rich'. Additionally, redistribution decreases as the polity adopts increasingly stringent super-majority rules.

Key Words: Redistribution, Inequality, Median Voter Theorem, Super-majority Rules, Core, Endogenous Factions.
JEL Codes: C7, D3, D7, H2

1 Introduction

Beginning with the seminal contributions of Romer (1975), Roberts (1977), and Meltzer and Richard (1981), a long literature has developed that explains the features of redistribution schemes as a consequence of the preferences of the median voter (see Epple and Romer (1991), Gans and Smart (1996), Besley and Coate (1997), Persson and Tabellini (1999), Moene and Wallerstein (2001), amongst many others). However, the logic of the median voter theorem requires that decisions be made by simple majority rule, and under most alternative decision procedures, the result does not obtain. And, in many contexts, decisions are indeed made by decision procedures other than simple majority rule. For example, in the United States, at both the federal level and in most states, decision-making requires the assent of majorities in both the House of Representatives and Senate, and the assent of the President (or Governor), unless two-thirds majorities exist in both chambers. Super-majority rules have become increasingly common, especially in the realm of fiscal policy.[1] And, of course, given its cloture rule, the United States Senate often operates under an effective super-majority rule.

Within the context of redistribution, models that assume decision-making by simple majority rule often generate stark predictions that are at odds with the data. For example, empirical evidence suggests that median income is not sufficient to predict the level of redistribution in a polity —income at other quantiles matter as well (see Karabarbounis (2011)). Similarly, standard median voter models of redistribution typically predict that redistribution should be increasing in the level of inequality (see Meltzer and Richard (1981)). However, as we discuss in more detail below, the evidence for this claim is strongly contested. These findings further motivate the need for a general approach to analyzing redistribution under decision procedures other than simple majority rule.

In this paper, we provide an analysis of redistribution under a broad class of decision procedures. Our approach retains the spirit of the median voter theorem in that we associate the equilibrium with a policy in the *core*.[2] However, unlike the case of simple majority, where

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[1]Indeed, 15 states require legislative super-majorities to raise taxes, or in some come cases, to pass a budget at all (see Rueben and Randall (2017)) In 2018, voters in Florida endorsed a ballot initiative requiring a super-majority to raise taxes, whilst voters in Oregon rejected a similar proposal. A similar ballot initiative was originally proposed in California, although was removed before the election. Moreover, proposals to institutionalize super-majority budget rules at the federal level via a constitutional amendment have been repeatedly introduced into Congress, albeit unsuccessfully (see H. J. Res. 111 (1998) and H. J. Res. 41 (2001).)

[2]The *core* is the set of unbeatable policies; i.e. those for which there does not exist some other policy that is strictly preferred by a decisive coalition of agents.

the core is uniquely the median voter's ideal policy, for most decision procedures the core generically contains (infinitely) many policies. Moreover, amongst this multiplicity, the core will often (and certainly whenever the decision procedure requires a super-majority) contain policies that are both above and below the median voter's ideal policy – providing little guidance as to even the qualitative effect of non-simple-majority rules.

This paper makes several contributions. First, we use a bargaining approach to select a unique robust policy from the multiplicity within the core. Whereas the core is an equilibrium concept arising out of cooperative game theory, we use the seminal non-cooperative bargaining model of Baron and Ferejohn (1989) to justify a particular core refinement. Furthermore, we demonstrate that this refinement has a simple and elegant characterization – it is the result of bilateral asymmetric Nash Bargaining between two voters (the *decisive* voters) whose identities depend on the decision rule, with endogenous bargaining weights that depend on the full income profile. Second, we show that equilibrium outcomes are sensitive to reversion policies, and that this relationship is non-monotonic – as disagreement outcomes are made more extreme, equilibrium policies become more moderate. Third, in the context of super-majority rules, we perform comparative statics on the decision rule by exploring the effect of varying the required super-majority. Under reasonable assumptions, taxation and redistribution are monotonically decreasing in the size of the required super-majority. Finally, we explore the effect of increasing inequality on the equilibrium demand for redistribution. When inequality increases, redistribution may either be higher or lower depending on incomes of the decisive voters (which are determined by the decision rule) and the shape of the income distribution. We provide conditions under which either of these outcomes may obtain; in brief, increasing inequality is likely to increase redistribution, unless at least one decisive voter has significantly above-average income —i.e. there is elite capture.

We present a simple public finance model in which there is a continuum of agents distinguished by their income. Redistribution is via a linear income tax (a proportional tax coupled with a uniform transfer to all agents), akin to the simplest model of a Universal Basic Income. For simplicity, we capture the dead-weight losses from taxation in reduced-form by assuming that redistribution is via a 'leaky-bucket' (see Dixit and Londregan (1998), Moene and Wallerstein (2001), amongst others). We micro-found these dead-weight losses in a model with elastic labor supply, in an online Appendix. Since the benefits of taxation are shared equally, but the costs fall more heavily on higher income earners, voters' most preferred income tax rates are decreasing in their incomes.

The *core* is the set of unbeatable, and thus stable, policies. Given a decision rule, it can be shown that the core is an interval of policies bounded by the ideal policies of two particular

agents, whom we call the *left* and *right decisive voters*, and whose identities are determined by the decision rule. (Under simple majority rule, the ideal policies of the left and right decisive voters coincide with the median voter's ideal, and so the core collapses to a singleton.)

In contexts where a failure to agree results in the implementation of a reversion policy, it is natural to focus attention on the subset of core policies that would themselves be chosen in preference to the reversion policy by a decisive coalition[3]. We call this the *core$^+$*. By construction, when the reversion policy is itself contained within the core, the core$^+$ is uniquely the reversion policy. However, we show that, when the reversion policy is outside the core, the core$^+$ generically remains an interval, and in fact, if the reversion policy is sufficiently extreme, the core$^+$ and core coincide.

We seek a refinement that selects a unique policy from within the core$^+$. To motivate our approach, we consider the outcomes that would obtain if policies were chosen under the seminal bargaining protocol of Baron and Ferejohn (1989). Crucial to the Baron & Ferejohn framework is a parameter δ that measures the (opportunity) cost of delay. For any $\delta < 1$, we show (analogously to Cardona and Ponsati (2011)) that the bargaining game admits a unique equilibrium in stationary strategies, characterized by an interval of policies that are socially acceptable. As $\delta \to 1$ (as delay becomes costless), this interval shrinks to a singleton, so that in the limit, a unique policy is chosen in equilibrium.

By contrast, when delay is exactly costless, the bargaining game generically admits a continuum of equilibria. Taking insights from Banks and Duggan (2000), we show that there is an equivalence between the core$^+$ and the set of equilibria of the bargaining game. For every core$^+$ policy, there is an equilibrium of the bargaining game (with $\delta = 1$) in which that is the unique policy that is proposed and accepted. Conversely, each bargaining equilibrium is associated with a particular core$^+$ policy.

These insights motivate our equilibrium refinement. Although the bargaining game admits multiple equilibria coinciding with the core$^+$ when $\delta = 1$, it admits a unique equilibrium for every $\delta < 1$. The equilibrium correspondence exhibits a failure of lower hemi-continuity at $\delta = 1$. Taking the limit as $\delta \to 1$, then, selects the unique and robust core$^+$ policy that survives as it becomes slightly costly to make counter-proposals. Our approach is analogous to Cho and Duggan (2009), who show that, under simple majority rule, this limit selects the median voter's ideal policy – thus providing bargaining micro-foundations for the median voter theorem. However, in contrast to Cho and Duggan (2009), for general decision rules,

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[3]The set of decisive coalitions is determined by the decision rule. We formalize these notions in the following section.

the selected policy generically does not coincide with the median's ideal. Rather, the selected policy is chosen as if by bilateral asymmetric Nash Bargaining between the left and right decisive voters, with endogenous weights that depend on the entire distribution of voters' incomes. Our approach, thus, provides a theoretical basis for the empirical finding that redistribution depends on the entire income profile, and not merely the median income (see Karabarbounis (2011)).

Our endogenous Nash Bargaining characterization admits the following interpretation: Although voters are heterogeneous in their preferences, they separate into two cohesive factions led by the left and right decisive voters. Policy is chosen as a consequence of asymmetric Nash Bargaining between the factional leaders, and their bargaining strengths depend on the sizes of their respective coalitions. Voters in turn understand that their factional choice affects the policy that results, and choose which faction to join, accordingly. The equilibrium coalitions are stable, in the sense that, given the policy that results, no voter would want to switch factions. Our result can thus be interpreted as a generalization of Duverger's Law, in that it micro-founds the emergence of two cohesive factions or 'parties'. However, these endogenous factions need not be equally sized, and will typically have non-median factional leaders.

Having characterized our refinement, we turn our attention to comparative statics in three dimensions. First, we highlight that the selected policy typically depends on the reversion policy that would have been implemented in the event of disagreement. (By contrast, under simple majority rule, the equilibrium policy is the median voter's ideal regardless of the reversion of policy.) Importantly, the relationship between reversion and equilibrium policies is non-monotone. As the reversion policy becomes more extreme, the equilibrium policy becomes more moderate, in the sense of being further from either boundary of the core. These results are consistent with the political logic of the 2013 budget sequester, which, by making disagreement more costly, was expected to generate a budget compromise in an otherwise gridlocked Congress. The results also suggest a particular logic for building sunset clauses into policy reforms that revert to an *ex ante* Pareto inferior policy.

Second, in the context of super-majority rules, we perform comparative static analysis on the decision rule itself by varying the size of the required super-majority $q \in [\frac{1}{2}, 1]$. As this requirement increases, we show that the level of redistribution decreases monotonically, provided that the income distribution is right-skewed and the reversion policy is 'low'.[4] Thus,

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[4]This result is far from obvious. As q increases, the core widens to include more policies both above and below the median voter's ideal. That the robust policy should be below the median's ideal is not immediately obvious.

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we provide a theoretical basis for the common wisdom that super-majority requirements will likely lead to lower taxation and redistribution.

Third, we analyze the effect of changing the income distribution of the legislature with a focus on the impact of increasing the inequality and skewness of the distribution. We show that, when inequality increases, redistribution may either be higher or lower depending on incomes of the decisive voters (which are determined by the decision rule) and the shape of the income distribution. If both decisive voters are poor (i.e. have income below some threshold quantile $\hat{\rho}$), then redistribution will increase, and vice versa. When rising inequality is coupled with greater skewness in the income distribution, this threshold income level is shown to be above the mean. Thus, consistent with Meltzer and Richard (1981), increasing inequality is likely to increase redistribution, unless at least one decisive voter has significantly above-average income —i.e. there is elite capture. Our model, therefore, suggests mechanisms that may reconcile the inconsistent empirical evidence on the effect on redistribution of increasing inequality.

In the context of super-majority rules, our theoretical results find support in the empirical literature. Knight (2000), Besley and Case (2003), and Lee (2014) find that super-majority budget rules are associated with a reduction in tax rates. Bradbury and Johnson (2006) and Bails and Tieslau (2000) find a similar results with regard to lower public welfare transfers and state expenditures, respectively. In a slightly different context, Heckelman and Dougherty (2010) demonstrate that there is an inverse relationship between the size of the majority requirement and tax rates on cigarettes and distilled spirits.

Nevertheless, as we previously noted, there has been little theoretical work that makes sense of these results, and indeed, this paper is the amongst the first to systematically study the effect of non-simple-majority rules on decision making *per se*, and on redistribution in particular. To the extent that existing papers have considered non-simple-majority rules, they have typically taken an *ad hoc* approach to equilibrium selection. For example, in the context of super-majority rules, several papers assume that the selected policy will be the one in the core[+] that is closest to the reversion (or status quo) policy. (See Gradstein (1999), Dal Bo (2006) and Riboni and Ruge-Murcia (2010). Gradstein (1999), in particular, presents a public finance model, similar to ours, except that his focus is on public goods provision rather than redistribution. In an earlier version of this paper, we showed that our results continue to hold in this alternative setting.) To rationalize this selection criterion, these papers consider suppose that policy is chosen as if through an ascending (resp. descending) auction-like format, in which, starting at the reversion policy, the committee considers a

sequence of proposals, each to increase (resp. decrease) the tax rate by (a further) ε. The procedure continues until a decisive coalition cannot be found to raise (or lower) taxes any further, and this is the selected policy.

Krehbiel (2010) has a different approach. In a model of the U.S. legislative system (including super-majority rule in the Senate, and a qualified executive veto), Krehbiel (2010) assumes that the selected policy will be the one in the core$^+$ closest to the median voter's ideal. The same dynamic appears in the seminal agenda setting model of Romer and Rosenthal (1978). Conversely, in their agenda-setting model of decision making by monetary policy committees, Riboni and Ruge-Murcia (2010) assume that the selected policy is the core$^+$ policy closest to the ideal policy of the committee chairman, rather than the median committee member.

Several (but not all) of these selection rules can be shown to be special cases of our refinement technique. However, our approach offers several distinct benefits. First, as the above examples make clear, the nature of equilibrium selection requires a motivating story that is particular to the specific context being studied, and this risks an *ad-hoc* approach to selection. By contrast, our approach provides a general approach to equilibrium selection that can be applied to any decision rule, in any context. Second, one could plausibly argue that the motivating stories themselves are suspect. Gradstein (1999), Dal Bo (2006) and Riboni and Ruge-Murcia (2010), for example, give far more prominence to the effect of *status quo* or reversion policies than can be rationalized by a bargaining story. Similarly, Krehbiel (2010) potentially gives undue influence to the median voter, especially given that, for decision rules other than simple majority, there is no theoretical basis for privileging the median voter.

Our theoretical results on the effect of changing the income distribution are also supported by the empirical literature. Since Meltzer and Richard's seminal analysis, many papers have attempted to confirm their finding that redistribution increases with increased inequality. Positive results in the cross-country context are documented in Meltzer and Richard (1983), Kristov, Lindert and McClelland (1992), and Persson and Tabellini (1994), Pogorelskiy and Traub (2017), amongst many others. On the other hand, there has been an abundance of work that questions these findings, beginning with Tullock (1983). Many cross-country analyses have produced largely null or negative results, including Lindert (1996) and Perotti (1996). Similar results are found for narrower samples, such as in the U.S. context (see Gouveia and Masia (1998), Moffitt, Ribar and Wilhelm (1998), and Rodrigiuez (1999)). Our model is able to accommodate both of these sets of results as our findings depend on the locations of both the decisive voters in the income distribution.

The literature on elite capture is also in conversation with our findings. Empirically, Cor-

valan, Querubin and Vicente (2016) show that the expansion of the franchise in the U.S. to poorer individuals was only associated with increased redistribution when less wealthy citizens were allowed to run for state office. Similarly, Mueller and Stratmann (2003) and Kenworthy and Pontusson (2005) suggest that higher low-income voter turnout leads to a positive relationship between inequality and redistribution. Li, Squire and Zou (1998) present evidence in support of the hypothesis that when democratic institutions are weak, wealthy elites will be able to sway policy in their favor both through economic power and direct political power. A similar argument is reflected in the theoretical literature by Acemoglu and Robinson (2006) and Acemoglu and Robinson (2008), who provide arguments that redistribution may remain low when a polity's income distribution becomes more unequal by means of an expansion of the franchise is that pre-existing, wealthy elites have an incentive to invest in institutions to increase their agenda setting power so they can remain politically decisive. In our model, elite capture can be thought of in terms of an effective change to the decision rule that assures that one of the decisive voters has well above-average income.

This paper contributes more broadly to the literature on bargaining in uni-dimensional policy spaces. Banks and Duggan (2006), in a framework that embeds ours as a special case, study bargaining games when disagreement entails reversion to a status quo. They show that such games always admit equilibria in no-delay, and that equilibrium proposals coincide with the status quo whenever the status quo is in the core. Cardona and Ponsati (2011) provide conditions for the equilibrium to be unique, which Parameswaran and Murray (2018) generalize. Predtetchinski (2011) shows that equilibrium proposals converge as $\delta \to 1$ and shows that the limit policy is the generalized root of a characteristic function. Under simple majority rule, Cho and Duggan (2009) show that this limit policy is simply the ideal policy of the median voter. In a search (as opposed to bargaining) framework, where proposals are generated from some exogenous process, Compte and Jehiel (2010) show that, under supermajority rule, the limit equilibrium policy is characterized by the solution to the *symmetric* bilateral Nash bargaining problem between the left and right decisive voters. By contrast, we show that when proposals arise endogenously, according to the rational choices of the players, then the solution corresponds to the *asymmetric* Nash bargaining solution with endogenous weights that depend on the distribution of voters' preferences. A similar result appears in a companion paper (see Parameswaran and Murray (2018)), albeit with important differences. Amongst them, disagreement does not result in the implementation of a reversion policy in that model, and so the distinction between the core and core$^+$ does arise. Additionally, given its grounding in the context of redistributive policies, this paper is able to explore several relevant comparative static results, especially with respect to the income distribution, that

are not present in the companion paper.

There is a more expansive literature that explores the connection between limit equilibria of bargaining games in certain multidimensional spaces and the multilateral Nash Bargaining solution. See, for example, Binmore, Rubinstein and Wolinsky (1986), Hart and Mas-Colell (1996), Miyakawa (2008), Laruelle and Valenciano (2008), amongst others. In an n player game, assuming payoffs are comprehensive[5], the limit equilibrium policy is the solution to $\max_x \prod_{i=1}^{n} u_i(x_i)^{p_i}$, where p_i is agent i's recognition probability. We note that two important differences between these results and our own. First, the assumption that preferences are comprehensive does not hold in our setting, or indeed in any setting where the median voter theorem might apply. Second, the limit policy depends on the preferences of all voters, not just the decisive voters. Thus, under simple majority rule, as $\delta \to 1$, the equilibrium policy will not generically select the median voter's ideal. These differences will hold more generally, for any decision rule.

The remainder of this paper is organized as follows: Section 2 introduces the model. Section 3 analyzes the equilibrium of the bargaining game and characterizes the limit equilibrium. Section 4 considers comparative statics on the reversion policy, the decision rule and the income distribution. Section 5 concludes.

2 Framework

2.1 Preferences

We present a public finance model analogous to Meltzer and Richard (1981). There is a unit mass of voters $i \in [0, 1]$. Let $y(i)$ be the income of agent i. For each $y \in \mathbb{R}_+$, we define the distribution function over incomes, $F(y) = \mathcal{F}(\{i \in [0, 1] | y(i) < y\})$ induced by the income function $y(i)$ and the measure $\mathcal{F}$ of voters on $[0, 1]$.[6] We assume F is continuously differentiable and thus admits a density f. Since the number of voters is large, F also represents the empirical distribution of incomes in the economy. Let $\bar{y} = \int y dF(y) < \infty$ be the average income.

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[5]Let $u = (u_1, ... u_n)$ be a feasible payoff vector, and let $u' \leq u$. The payoff set is comprehensive, if u' is also feasible.

[6]To ensure that all these objects are well defined, we restrict attention to coalitions in the Borel σ-algebra on $[0, 1]$, and require that the function $y : [0, 1] \to \mathbb{R}$ be measurable.

The government can levy a proportional income tax $\tau \geq 0$ that finances a uniform lump-sum transfer T to each individual. Agents supply their labor inelastically, so that each agent's pre-tax income is unaffected by the tax policy. However, taxes are collected via a leaky bucket (see Moene and Wallerstein (2001) and Dixit and Londregan (1998), amongst others), which implies dead-weight losses.[7] Let $e(\tau) \in [0,1]$ be the effective tax rate.[8] Then, if the government levies a proportional labor tax $\tau \in [0,1]$, it actually receives only $e(\tau)$ for each dollar of income taxed. We assume that e is strictly concave, $e'(0) = 1$ and $e'(\bar{\tau}) = 0$ for some $\bar{\tau} \in (0,1)$. Together, these imply that $e(\tau) < \tau$ whenever $\tau > 0$. In the absence of dead-weight losses, a tax rate of τ would generate revenue of $\tau \int y dF(y) = \tau \bar{y}$. Instead, the government's revenue is $R(\tau) = e(\tau)\bar{y} < \tau\bar{y}$.

The revenue function $R(\tau)$ represents the Laffer curve. By construction, $\bar{\tau}$ is the tax rate that maximizes government revenue; i.e. the tax rate associated with the peak of the Laffer curve. Given the strict concavity of e, government revenue is increasing in the tax rate when $\tau \in (0, \bar{\tau})$ and it is decreasing when $\tau \in (\bar{\tau}, 1)$. Naturally, in equilibrium, we should never expect $\tau > \bar{\tau}$.

The consumption of a voter with pre-tax income y is $T + (1-\tau)y$. Voters have expected utility preferences over consumption, represented by a continuous and concave utility index u, where $u' > 0$ and $u'' \leq 0$.

Government transfers are financed using tax revenues, and the government's budget is assumed to be in balance. (Accordingly, we ignore spending by the government on public goods[9], or the possibility of debt financing.) Given the government budget constraint, and since there is a unit measure of agents, the government's revenue is also the size of the transfer T that the government can give each voter. Hence, at tax rate τ, a voter with pre-tax income y has consumption $c(\tau, y) = e(\tau)\bar{y} + (1-\tau)y$.

Let $v(\tau, y)$ represent the preferences over tax policies of an agent with income y, taking the government's budget constraint as given. We have: $v(\tau, y) = u\left(e(\tau)\bar{y} + (1-\tau)y\right)$. Since e is strictly concave, then $c(\tau, y)$ is strictly concave in τ for each y. Then, since u is increasing

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[7]Our reduced-form approach is mostly for simplicity, and enables us to focus attention on the novel feature of our paper —equilibrium selection via bargaining. We present a micro-founded version of the model in an online Appendix, available at http://ww3.haverford.edu/economics/faculty_and_staff/gparames/research/.

[8]In the online Appendix, we connect the reduced-form dead-weight losses to those arising from a micro-founded model with endogenous labor supply. Briefly, let $\epsilon_\tau(\tau, \bar{y}) = \frac{\partial \bar{y}}{\partial \tau} \cdot \frac{\tau}{\bar{y}}$ denote the tax elasticity of average income in a model with elastic labor supply. Then setting $e(\tau) = \tau + \int_0^\tau \epsilon_\tau(t, \bar{y})dt$ causes the dead-weight loss to have the same marginal behavior in both the structural and reduced-form models.

[9]In an earlier draft, we showed that our results continued to hold exactly, in a model with public goods analogous to Gradstein (1999).

and concave, $v(\tau, y)$ is strictly concave in τ for each y, and so preferences are single-peaked. We have:

$$v_\tau(\tau, y) = [e'(\tau)\overline{y} - y]u'(c). \tag{1}$$

Let $\tau(y) = \arg\max_{\tau \in [0,1]} v(\tau, y)$ denote the most preferred tax rate of an agent with income y. Since v is strictly concave, this most preferred policy satisfies the first order condition:

$$e'(\tau)\overline{y} - y \leq 0 \tag{2}$$

where the condition holds with equality unless the optimum is at $\tau = 0$. Notice that equation (2) is analogous to equation (13) in Meltzer and Richard (1981), which defines the optimal tax rate for agents in their setting. We can explicitly characterize the optimal tax rate for a given voter by[10]:

$$\tau(y) = \begin{cases} [e']^{-1}\left(\frac{y}{\overline{y}}\right) & y < \overline{y} \\ 0 & y \geq \overline{y} \end{cases}$$

A marginal increase in taxes has two effects; it increases the size of the transfer from the government whilst reducing the voter's take home pay. The ideal tax rate for a given agent is the one that appropriately balances these competing effects. An agent with zero income would ideally have taxation at the peak of the Laffer curve (i.e. $\tau = \overline{\tau}$), since this maximizes the transfer received at no cost to their post-tax income. As income increases, the lost earnings become more salient and so the voter's ideal tax rate decreases. Indeed, by the implicit function theorem, we have: $\frac{\partial \tau}{\partial y} = \frac{1}{e''(\tau(y))} < 0$. Moreover, all voters with income above the mean would ideally prefer zero redistribution.

The concavity assumption disciplined the preferences over consumption/tax plans of each individual voter. We supplement this with an additional requirement that disciplines the behavior of preferences across individuals. Formally, we require that preferences satisfy the Spence-Mirrlees condition:

$$v_{\tau y}(\tau, y) = -u'(c) + [e'(\tau)\overline{y} - y](1 - \tau)u''(c) \leq 0$$

. The Spence-Mirrlees condition requires that the marginal utility of taxation is monotone in agents' income. It implies that, if an agent approves of a particular tax increase, so will all agents with strictly lower income, and if an agent disapproves of that tax increase, so will all agents with strictly higher income.[11]

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[10]$e'(\cdot)^{-1}$ is well defined since, by the strict concavity of e, e' is strictly decreasing.

[11]To see this, take any two redistributive policies τ_1 and τ_2 with $\tau_1 < \tau_2$. Then: $\frac{\partial}{\partial y}[v(\tau_2, y) - v(\tau_1, y)] =$

Notice that, by the concavity of u, the Spence-Mirrlees condition is automatically satisfied whenever $y \leq e'(\tau)\overline{y}$. Hence, adding this requirement only potentially has bite if $y > e'(\tau)\overline{y}$. Let $R(c) = -c\frac{u''(c)}{u'(c)}$ be the coefficient of relative risk aversion. It is easily verified that the following assumption on preferences over consumption guarantees that the Spence-Mirrlees condition holds:

Assumption 1.
$$R(c^*) \leq \frac{(1-\tau)\,y + e(\tau)\,\overline{y}}{(1-\tau)\,y - (1-\tau)\,e'(\tau)\,\overline{y}}$$

for every $\tau \in [0, \overline{\tau}]$ and every y for which $y > e'(\tau)\overline{y}$ (i.e. for which the denominator is positive).

It suffices that the degree of relative risk aversion of higher income agents not be too large. The condition is satisfied for commonly used classes of preferences. For example, if preferences over consumption are CRRA (with coefficient of relative risk aversion θ), then the condition is satisfied for any $\theta \leq 1$, which includes log utility as a special case.

Finally, as we establish in the online Appendix, in a model with endogenous labor supply, assuming that the Spence-Mirrlees condition holds is equivalent to assuming that no agent increases her work effort following a tax increase. Agents with $y \leq e'(\tau)\overline{y}$ would never do so, because the substitution and wealth effects associated with a tax increase (including the increased governmental transfer) both push in the direction of deterring work effort. By contrast, if $y > e'(\tau)\overline{y}$, the marginal loss in earned income is larger than the marginal gain from the transfer, and so the wealth effect pushes in the direction of working more. The Spence-Mirrlees condition ensures that this wealth effect does not overwhelm the substitution effect.[12]

2.2 Social Decision Making

Let $C \subset [0, 1]$ denote a coalition of voters, and let d be a decision rule. A coalition is *decisive* if the support of all agents in C is sufficient to have a policy adopted. Let $\mathcal{C}(d)$ denote the set of decisive coalitions under decision rule d. For example, in the American legislative system, a coalition is decisive if it contains either: (i) the president, (at least) a simple majority of

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$\frac{\partial}{\partial y}\int_{\tau_1}^{\tau_2} v_\tau(\tau, y)d\tau = \int_{\tau_1}^{\tau_2} v_{\tau y}(\tau, y)d\tau \leq 0.$

[12]The assumption does not rule out a backward-bending labor supply curve. However, it insists that labor supply cannot bend back too far. Put differently, whilst the wage elasticity of labor supply may be negative, the tax elasticity of labor supply may not be positive.

legislators in the House of Representatives, and (at least) a three-fifths majority of Senators; or (ii) (at least) a two-thirds majority of legislators in each chamber.

Our framework admits a broad range of decision rules, subject to the requirement that $\mathcal{C}(d)$ is (i) *monotone* ($C \in \mathcal{C}$ and $C \subset C'$ implies $C' \in \mathcal{C}$), and (2) *proper* ($C \in \mathcal{C}$ implies $[0,1] \setminus C \notin \mathcal{C}$). Thus, our framework is consistent with many commonplace decision procedures, including:

- q-majority rule for any $q \geq \frac{1}{2}$ ($\mathcal{C} = \{C \subseteq [0,1] \,|\, \mathcal{F}(C) \geq q\}$), which includes simple majority rule ($q = \frac{1}{2}$) and unanimity rule ($q = 1$) as special cases.

- *coalitional* rules ($\cap_{C \in \mathcal{C}(d)} C \neq \emptyset$) that endow a subset of players with an absolute veto. For example, the U.N. Security Council rule requires a majority of all 15 members, as well as the support of each of the five permanent members.

- *oligarchic* rules ($\cap_{C \in \mathcal{C}(d)} C \in \mathcal{C}(d)$), which include *dictatorial* rules as a special case.

2.3 The Core

Suppose redistributive policies are chosen according to decision rule d, such that to be implemented, a policy requires the assent of a decisive coalition. The *core* is the set of stable policies – the set of policies for which there does not exist another policy that is strictly preferred by a decisive coalition. Under simple majority rule, we know that the core is uniquely the ideal policy of the median voter, τ_{med} (see Black (1948) and Downs (1957)).

Consider any generic (monotone and proper) decision rule d. Let $y_L = \sup_{C \in \mathcal{C}(d)} \inf_{i \in C} y(i)$ and $y_R = \inf_{C \in \mathcal{C}(d)} \sup_{i \in C} y(i)$ be the incomes of the *left* and *right decisive agents*, respectively.[13] A coalition C cannot be a decisive coalition if its lowest income earner has income larger than y_L or its highest income earner has income below y_R. Under q-majority rule, $y_L = F^{-1}(1-q)$ and $y_R = F^{-1}(q)$, where $q \in [\frac{1}{2}, 1]$. Let $\tau_L = \tau(y_L)$ and $\tau_R = \tau(y_R)$ be the ideal tax rates for the left and right decisive players, respectively. The assumption that d is proper ensures that $y_L \leq y_R$ and so $\tau_L \geq \tau_R$.

The core is the set of policies in the interval $[\tau_R, \tau_L]$. Since, generically, $\tau_R < \tau_L$, there will typically be a multiplicity of possible 'equilibrium' policies. Furthermore, for many rules,

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[13]Restricting attention to super-majority rules, Cardona and Ponsati (2011) refer to them as the left and right *boundary* players.

$\tau_R < \tau_{med} < \tau_L$, and so the core contains policies that are both higher and lower than the median voter's ideal policy. Absent a selection criterion, we cannot predict which policy will prevail for generic decision procedures. Moreover, from a comparative static perspective, for generic decision procedures, it is indeterminate whether redistribution will be higher or lower than under simple majority rule.

Now suppose the polity's failure to actively adopt a policy results in the implementation of a 'reversion' policy, τ_0. (In some contexts policy may revert to the 'status quo', but it may also revert to some other pre-specified outcome. For example, the failure of Congress to pass a budget may result in dramatic cuts to government spending, as in the 2013 'sequester', or even a cessation of government spending altogether, when the government shuts down. Alternatively, the failure to agree may result in a higher tax rate, for example if the government fails to agree to extend temporary tax cuts.) Then, there is a natural way to refine the core: limit attention to policies in the core (i.e. 'stable' policies) that would themselves be chosen by some decisive coalition in preference to the reversion policy. We refer to this set as the $core^+$.

The $core^+$ has a simple characterization; it is the subset of core policies that are weakly preferred by both decisive voters to the reversion policy. The distinction between the core and $core^+$ is analogous to the distinction between the set of Pareto optima and the contract curve; the latter is the subset of Pareto optima that are also Pareto improvements given the agents' endowments. Straightforwardly, if the reversion policy is contained in the core (i.e. $\tau_0 \in [\tau_R, \tau_L]$), then $core^+ = \{\tau_0\}$; the only core policy that is weakly preferred by a decisive coalition to the reversion policy is the reversion policy itself. By contrast, if the reversion policy lies outside the core, then the $core^+$ will be an interval whenever the core is. Moreover, if the reversion policy is sufficiently extreme, then every policy in the core will be an improvement over the reversion policy for some decisive coalition, and so the core and $core^+$ will coincide.

3 Equilibrium Selection

In this section, we develop a procedure for selecting a robust equilibrium policy whenever the $core^+$ contains a multiplicity. We begin by analyzing the redistributive outcomes that would result if chosen as a consequence of bargaining within a committee or legislature. We will subsequently demonstrate that we can make a compelling selection from amongst the set of $core^+$ policies by taking a particular limit equilibrium of the bargaining game.

3.1 The Bargaining Protocol

The bargaining protocol is the standard procedure in Baron and Ferejohn (1989) and Banks and Duggan (2000). There are potentially infinitely many bargaining rounds. In a given round of bargaining, a voter is randomly recognized to propose a policy. Let $P(y)$ be the probability that the recognized proposer has income less than y, and suppose $P(y)$ admits a density $p(y)$. In the special case that $P(y) = F(y)$, voters are recognized to propose with equal probability. However, the framework easily accommodates unequal recognition probabilities, which might, for example, capture the idea that richer voters tend to exert more influence in policy-making than poorer voters (see Benabou (2000)). After observing the proposal, all players simultaneously vote to either accept or reject the proposal. Acceptance requires that the proposal receive the assent of a decisive coalition of voters. If so, the policy is implemented, and the bargaining game ends. In the event of disagreement, with probability $\delta \in [0, 1)$, the bargaining game continues, and a new proposer is recognized to make an offer. However, with probability $1 - \delta$, bargaining terminates exogenously, and the reversion policy τ_0 is implemented. Thus, δ parameterizes the opportunity cost of delay in terms of the likelihood that bargaining will terminate.[14]

A strategy for a voter with income y is a pair $(t(y), A(y))$, where t is the tax rate proposed whenever a type y voter is recognized[15], and $A \subset [0, 1]$ is the set of tax rates that such a voter will accept. We solve for stationary sub-game perfect equilibria in weakly undominated strategies. The weak undominance requirement implies that each agent votes as if they were pivotal (i.e. they only support proposals that they weakly prefer to the continuation game).[16]

We say an equilibrium is in *no-delay* if there will be immediate agreement in equilibrium, regardless of the identity of the proposer. We say that an equilibrium is *static* if the implemented policy is unchanging across periods, even if immediate agreement is not reached. In a framework that embeds ours, Banks and Duggan (2006) establish (see Theorems 1,4 and 7), that there always exists an equilibrium in no-delay, and that, whenever the reversion policy lies outside the core, equilibria must be in no-delay. Furthermore, they show

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[14]The bargaining framework admits an alternative interpretation in which, following disagreement, players adjourn and the reversion policy is implemented in the current period. Players then reconvene in the following period to continue bargaining. Under this interpretation, δ is the discount factor.

[15]To clarify the notation: $\tau(y)$ is the ideal tax rate of a type-y voter, whereas $t(y)$ is the equilibrium tax proposal for that voter.

[16]Since every voter is an atom, no agent's vote can sway the outcome of an election. If so, any voting strategy can be sustained as an equilibrium, since no single agent's vote matters. The weak-dominance refinement rules out perverse equilibria of this sort, by requiring agents to vote for their preferred alternative, even though no agent's choice, in isolation, can affect the policy outcome.

that whenever there is an equilibrium with delay, it must be a static equilibrium. If so, the policy implemented in any equilibrium must coincide with the policy chosen in a no-delay equilibrium. Hence, we focus on no-delay equilibria, and this is without important loss of generality.

3.2 Equilibrium

In this section we characterize the bargaining equilibrium. Since these results have antecedents in the existing literature (see Cho and Duggan (2003), Cardona and Ponsati (2011), Predtetchinski (2011), amongst others), our treatment will be brief. Let $t(y)$ be the equilibrium proposal of a type-y agent. The expected equilibrium utility of a type-y agent is $V(y) = \int_0^\infty v(t(z), y) dP(z)$. Hence, y will accept a proposal t if:

$$ v(t, y) \geq (1 - \delta) v(\tau_0; y) + \delta \int_0^\infty v(t(z), y) dP(z) $$

Since v is strictly quasi-concave in τ for each y, then the acceptance sets for each agent must be an interval. Let $A(y) = [\underline{t}(y), \bar{t}(y)]$, where $\underline{t}(y)$ and $\bar{t}(y)$ are the left and right certainty equivalents given the lottery over policies induced by the continuation game. For each y, we have:

$$ \underline{t}(y) = \min\{t \geq 0 \mid v(t, y) \geq (1 - \delta) v(\tau_0, y) + \delta V(y)\} $$
$$ \bar{t}(y) = \max\{t \leq 1 \mid v(t, y) \geq (1 - \delta) v(\tau_0, y) + \delta V(y)\} $$

Let $E[t] = (1 - \delta)\tau_0 + \delta \int_0^\infty t(y) dP(z)$ be the expected policy in the continuation game. Since v is concave in τ, $v(E[t], y) \geq (1 - \delta) v(\tau_{sq}, y) + \delta V(y)$ for all y, and so $\underline{t}(y) \leq E[t] \leq \bar{t}(y)$.

Let $A_C = \cap_{i \in C} A(y(i))$ be the set of policies that will be accepted by decisive coalition $C \in \mathcal{C}(d)$, and let $A = \cup_{C \in \mathcal{C}(d)} A_C$ be the set of policies that will be accepted by *some* decisive coalition. Since each $A(y)$ is an interval and $E[t] \in A(y) \ \forall y$, A must also be an interval. We have $A = [\underline{t}, \bar{t}]$.

Given the Spence-Mirrlees condition, we can show that any proposal not accepted by the right decisive voter will not be accepted by any player whose income is even higher. Similarly, any proposal not accepted by the the left decisive voter, will not be accepted by any voter whose income is even lower. Equilibrium coalitions must be connected. Hence, to be socially

acceptable, it is necessary and sufficient that a proposal be acceptable to both the left and right decisive agents. Hence, the social acceptance set is $A = [\underline{t}(y_L), \bar{t}(y_R)]$. The set of socially acceptable proposals are those that are not too low from the perspective of the left decisive voter and not too high from the perspective of the right decisive voter. With this discussion in mind, we have the following result:

Proposition 1. *The bargaining game admits a unique stationary equilibrium in no-delay, characterized by a pair of thresholds $\underline{t}(y_L)$ and $\bar{t}(y_R)$, where:*

1. *Equilibrium proposals are given by[17]:*

$$
t(y) = \begin{cases} \bar{t}(y_R) & y \leq \tau^{-1}\left(\bar{t}(y_R)\right) \\ \tau(y) & y \in \left(\tau^{-1}\left(\underline{t}(y_R)\right), \tau^{-1}\left(\bar{t}(y_L)\right)\right) \\ \underline{t}(y_L) & y \geq \tau^{-1}\left(\underline{t}(y_L)\right) \end{cases}
$$

2. *The acceptance sets are given by: $A(y) = \left[\underline{t}(y), \bar{t}(y)\right]$, where:*

$$
\underline{t}(y) = \min\left\{t \geq 0 \,\middle|\, v(t, y) \geq (1 - \delta) v(\tau_0, y) + \delta V\left(\underline{t}(y_L), \bar{t}(y_R), y\right)\right\}
$$
$$
\bar{t}(y) = \max\left\{t \leq 1 \,\middle|\, v(t, y) \geq (1 - \delta) v(\tau_0, y) + \delta V\left(\underline{t}(y_L), \bar{t}(y_R), y\right)\right\}
$$

and $V\left(\underline{t}(y_L), \bar{t}(y_R), y\right) = F\left(\tau^{-1}\left(\bar{t}(y_R)\right)\right) v\left(\bar{t}(y_R), y\right) + \int_{\tau^{-1}\left(\bar{t}(y_R)\right)}^{\tau^{-1}\left(\underline{t}(y_L)\right)} v\left(\tau(z), y\right) dF(z) +$ $\left[1 - F\left(\tau^{-1}\left(\underline{t}(y_L)\right)\right)\right] v\left(\underline{t}(y_L), y\right).$

Any agent whose ideal policy lies in the social acceptance set will simply propose their ideal policy whenever they are recognized to propose. All remaining agents will propose one of the end-point policies —whichever is closest to their ideal policy. Furthermore, Proposition 1 pins down the boundaries of the social acceptance set as a consequence of the preferences of the left and right decisive voters.

The set of socially acceptable policies naturally depends on the reversion policy that is implemented in the event of disagreement. If the reversion policy is contained in the core (i.e. if $\tau_0 \in [\tau_R, \tau_L]$, which implies that the core$^+$ is simply $\{\tau_0\}$), then the only socially acceptable policy is the reversion policy (see Theorem 7 in Banks and Duggan (2006)). By construction, there is no decisive coalition that would agree to replace τ_0 with some other policy, and so intuitively, there cannot be any other policies that are socially acceptable.

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[17] $\tau^{-1}(t)$ is well defined for any $t \in (0, \bar{\tau}]$. For $t = 0$, define $\tau^{-1}(t) = \inf\{y \geq 0 | \tau(y) = 0\} = \bar{y}$

By contrast, if the reversion policy is not contained in the core, then for every $\delta < 1$, the social acceptance set $A = [\underline{t}, \overline{t}]$ is a true interval, so that $\underline{t} < \overline{t}$. Thus, when delay is costly, there are a range of policies that will potentially be implemented in equilibrium. To see why, note by the preceding discussion that, if the expected policy, $E[t]$, is proposed, then it will receive unanimous support. Since the reversion policy is outside the core, the continuation lottery must be non-degenerate, and so (since $\delta < 1$) there must be some neighborhood around $E[t]$ in which proposals will also be unanimously accepted. Hence, starting from $E[t]$, the proposer can offer a policy slightly closer to her ideal without losing unanimous support. In fact, she can afford to pull the policy in her desired direction until she either arrives at her ideal policy, or the support of one of the decisive voters would be lost. In the latter case, she would simply propose the most extreme policy that the decisive voter would accept.

Hence, when the reversion policy lies outside the core, the social acceptance set is generically an interval, and so there is a range of policies that may arise in equilibrium, depending on the identity of the proposer. By contrast, when the reversion policy lies within the core, the social acceptance set collapses to a singleton.

The parameter δ captures the (opportunity) cost of delay. Because bargaining terminates with probability $1 - \delta$ when a policy is rejected, voters face an opportunity cost from rejecting the current proposal in terms of future bargaining opportunities forgone. It is this opportunity cost that empowered the proposer to pull the policy away from the mean proposal and towards her preferred policy. Thus, δ also parameterizes the proposer's relative degree of agenda control.

Proposition 2. *The equilibrium proposals converge as $\delta \to 1$. Formally, $A \to [t^*, t^*]$ as $\delta \to 1$ (i.e. $\lim_{\delta \to 1} \underline{t}(y_L) = t^* = \lim_{\delta \to 1} \overline{t}(y_R)$).*

Proposition 2 is analogous to Theorem 3.6 from Predtetchinski (2011). The intuition is precisely as in the discussion above. Proposers are able to exercise a degree of agenda control in the bargaining game to the extent that costly delay creates a disincentive for other players to reject non-mean proposals, in favor of the continuation game. As delay becomes costless, a blocking coalition can always be found who would rather face the continuation game than implement any non-mean proposal. This forces all proposers to make identical proposals. In the limit as $\delta \to 1$, not only is the equilibrium unique, but a unique policy will be proposed by all players in equilibrium.

Things are different, however, when delay is exactly costless (i.e. $\delta = 1$). In this world, there are, in fact, multiple equilibria. Similar to the limiting equilibrium (as $\delta \to 1$), each

equilibrium is associated with a single policy that is proposed and accepted by all agents. However, there is now an equivalence between the core$^+$ and the set of equilibria. Every core$^+$ policy can be sustained as a bargaining equilibrium when $\delta = 1$, and every bargaining equilibrium selects a core$^+$ policy.[18]

3.3   Equilibrium Selection and the Nash Bargaining Solution

We now return to the question of equilibrium selection from the core$^+$. Notice that, when $\delta = 1$, there are potentially a continuum of equilibria of the bargaining game, coinciding with core$^+$. By contrast, for any $\delta < 1$, there is a unique equilibrium, and these equilibria converge to a unique policy as $\delta \to 1$. The equilibrium correspondence exhibits a failure of lower-hemicontinuity at $\delta = 1$, and this failure presents a natural candidate for an equilibrium refinement. Although every policy in the core$^+$ can be sustained as an equilibrium when $\delta = 1$, only one of these equilibria remains for δ slightly below 1. This limit equilibrium policy is the unique robust policy in the core$^+$ that survives the introduction of small positive costs to making counter-proposals. We take this to be a focal policy amongst the many within the core$^+$.

We stress that our approach to equilibrium selection is not *ad hoc*. We do not focus on the bargaining limit simply because it selects a core$^+$ policy; after all, one can imagine other criteria that also guarantee a selection from the core$^+$. Rather, our approach is justified by the equivalence of the core$^+$ to the bargaining equilibria when $\delta = 1$. The tight connection between the bargaining game and the core motivates our use of the bargaining approach, and the failure of lower-hemicontinuity at $\delta = 1$ recommends its use as a selection criterion.

The limit equilibrium policy additionally has a simple and elegant characterization that further recommends its use. First, some notation. Denote by $\Delta v(t, y) = v(t, y) - v(\tau_0, y)$ the utility improvement for voter y of any policy t over the reversion policy. Take any $\phi \in [0, 1]$, which represents the bargaining weight of the left decisive voter. Let $B(\phi, y_L, y_R)$ denote the solution to the asymmetric bilateral Nash Bargaining problem between the left and right decisive voters:

$$B(\phi, y_L, y_R) = \arg\max_t [\Delta v(t, y_L)]^\phi [\Delta v(t, y_R)]^{1-\phi}$$

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[18]Banks and Duggan (2000) study bargaining games under a commonly used alternative framework, where each agent perceives disagreement as being worse than any feasible policy. In that setting, they show that, when delay is costless, the set of bargaining equilibria coincides with the entire core. Analogous logic establishes the core$^+$ equivalence in our setting.

It is straightforward to show that B is strictly increasing in ϕ whenever $y_L < y_R$. Notice that, for any ϕ, $B(\phi, y_L, y_R) \in core^+$, since Nash Bargaining always selects Pareto optimal outcomes that are improvements for both players. In particular, if $\tau_0 \in [\tau_R, \tau_L]$, then the only candidate solution is τ_0, since any other policy would be worse than the reversion policy for at least one of the decisive voters.

In what follows, we make explicit the dependence of the equilibrium and limit policies on the decision rule d. In particular, we denote the incomes of the left and right decisive voters by $y_L(d)$ and $y_R(d)$.

Proposition 3. *The limit equilibrium policy $t^*(d)$ is characterized by the following system:*

$$t^*(d) = B\left(\phi^*(d), y_L(d), y_R(d)\right)$$
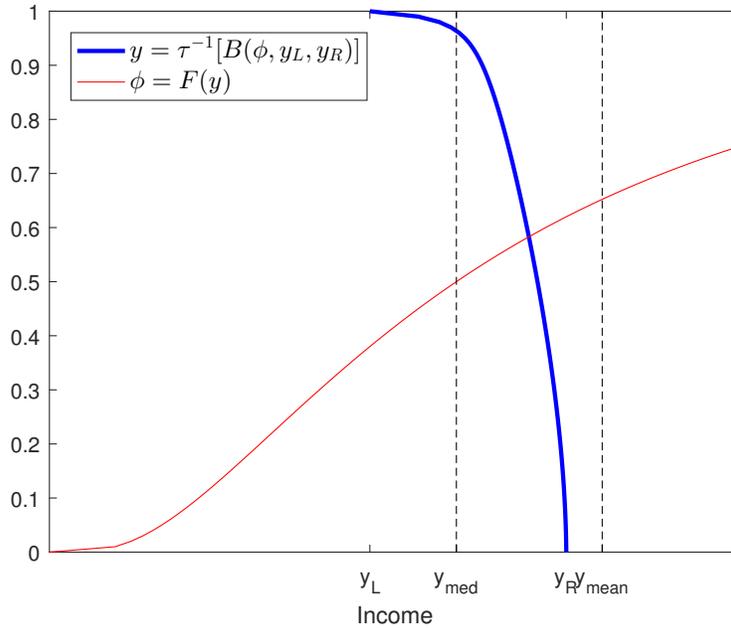$$\phi^*(d) = P\left(\tau^{-1}(t^*(d))\right)$$

Figure 1: Income of the Pivotal Voter under 60 percent super-majority rule. The income distribution is assumed log-normal, with variance calibrated to the U.S. Gini coefficient. Voters are recognized to propose with equal probability. The thick line represents the identity of the 'pivotal' voter for different values of ϕ, given the decision rule. The reversion policy is zero redistribution. In equilibrium, the decisive voter is richer than the median, and the left faction represents more than half of voters.

Proposition 3 shows that our refinement has a simple and elegant characterization. In the limit, the equilibrium tax rate is the consequence of asymmetric Nash Bargaining between

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the left and right decisive voters, with endogenous bargaining weights that depend on the distribution of recognition probabilities. In the limit, it as if the voters separate into two distinct factions, and that the bargaining outcome is the result of Nash bargaining between the factional leaders. Moreover, since the relative likelihood that a proposal emanates from a given faction is determined by the number of agents in that faction, the bargaining weights are proportional to the recognition probability weighted size of each faction.

We see this in Figure 1. The thick line shows the Nash Bargaining outcome for arbitrary bargaining weight ϕ. (In fact, it displays the income of the 'pivotal' voter – the agent for whom the chosen policy is optimal.) As ϕ increases from 0 to 1, the Nash Bargaining policy increases from R's ideal tax rate to L's, which implies that the 'pivotal' voter becomes poorer. The thin line is the cumulative distribution of income, assumed log-normal and with variance calibrated to the U.S. Gini coefficient. If voters are recognized to propose with equal probability (i.e. $P(y) = F(y)$), then this also represents the recognition-probability weighted size of faction L, when the pivotal voter has income y. The limit equilibrium is characterized by the intersection of these curves. As the diagram makes clear, the pivotal voter need not be the median income earner, and the equilibrium coalitions need not be equally sized.

To make sense of this result, take any $\delta < 1$, and let $\left[\underline{t}_L(\delta), \bar{t}_R(\delta)\right]$ be the associated social acceptance set. Then all agents with $y \leq \tau^{-1}\left(\bar{t}_R(\delta)\right)$ will propose $\bar{t}_R(\delta)$, all agents with $y \geq \tau^{-1}\left(\underline{t}_L(\delta)\right)$ will propose $\underline{t}_L(\delta)$, and all remaining agents will propose their ideal policies. Hence, a measure $P\left(\tau^{-1}\left(\bar{t}_R(\delta)\right)\right)$ will behave as a faction and propose $\bar{t}_R(\delta)$, and a measure $1 - P\left(\tau^{-1}\left(\underline{t}_L(\delta)\right)\right)$ will behave as a faction and propose $\underline{t}_L(\delta)$. Now, as $\delta \to 1$, since $\bar{t}_R(\delta) - \underline{t}_L(\delta) \to 0$, the measure of agents who propose their own ideal policy is squeezed to zero. In the limit, it is as if there are simply two cohesive factions, led by the decisive voters, who bargain over the equilibrium policy.

A different way to conceive of this result is to consider the voters' decisions about which faction to join. Take a (connected) ϵ measure of agents. Those agents understand that joining one faction over the other increases the bargaining weight of the former, which pushes the equilibrium policy towards the ideal policy of the leader of the faction joined. Voters rationally make their factional choices, anticipating the policy that will follow. Hence, the equilibrium policy and coalitions are both determined endogenously. The policy identified in Proposition 3 is the unique policy that is equilibrium consistent, in the sense that the policy induces voters to separate into particular factions, and the bargaining weights implied by those factions cause the bargaining between factional leaders to select the equilibrium policy. In equilibrium, no (ϵ mass of) voters could do better by switching factions.

Our result suggests an interesting analogue to Duverger's Law, that under plurality rule, the stable number of parties is two. Along with median voter logic, this suggests that voters should divide into two equally sized factions that both advocate for the policy preferred by the median voter. For generic decision rules, our results suggest that Duverger's Law should continue to hold. However, these factions need no longer be equally sized. In Section 4, we show that in the context of super-majority rules, we may reasonably expect two parties: a larger party of the poor and a smaller party of the rich. Moreover, these parties will be credibly led by agents who themselves have non-median preferences.

Meltzer and Richard (1981) identify the equilibrium policy with the ideal policy of a pivotal voter, who, under simple majority rule, is the median income earner. More generally, we identify the pivotal voter as the one whose ideal policy coincided with the equilibrium policy (i.e. $y^* = \tau^{-1}(t^*)$.) Such a voter is pivotal in the sense of being indifferent between joining either faction. Given the Nash Bargaining framework, we can also interpret the pivotal voter as the one whose income is a particular 'average' of the incomes of the left and right decisive voters. To see this, note that, by construction, $v_\tau(t^*, y^*) = 0$ and so, by the first order condition that characterizes the Nash bargaining solution:

$$(1 - \phi^*)\frac{v_\tau(t^*, y_L)}{\Delta v(t^*, y_L)} + \phi^*\frac{v_\tau(t^*, y_R)}{\Delta v(t^*, y_R)} = \frac{v_\tau(t^*, y^*)}{\Delta v(t^*, y^*)}$$

Then, letting $g(\cdot) = \frac{v_\tau(t^*, \cdot)}{\Delta v(t^*, \cdot)}$ demonstrates that y^* is the generalized-g weighted average of y_L and y_R.[19] Hence, for generic decision rules, rather than being the voter with the median income, the pivotal voter is the one whose income is a (generalized) weighted average of the incomes of the left and right decisive voters, where the weights depend on the sizes of each faction.

4 Comparative Statics and Policy Implications

We now turn our attention to various comparative statics. First, we consider the effect of changing the reversion policy on the equilibrium level of redistribution. Second, in the context of super-majority rules, we take comparative statics on the decision rule itself, by

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[19]For any increasing function g, the generalized-g weighted mean of two numbers x_1 and x_2, with weights ϕ and $1 - \phi$ is a number x_m satisfying $g(x_m) = \phi g(x_1) + (1 - \phi)g(x_2)$. Several ex amples of generalized means are familiar. Setting $g(x) = x$ gives the arithmetic mean; setting $g(x) = \log x$ gives the geometric mean, and likewise $g(x) = \frac{1}{x}$ gives the harmonic mean. Moreover, if $g(x) = u(x)$ where u is an expected utility index, then the generalized mean is the certainty equivalent.

considering the effect of changing the size of the required super-majority. Finally, we consider the effect of inequality and skewness in the income distribution on the demand for redistribution.

4.1 Changing the Reversion Policy

Fix some decision rule d, and the associated left and right decisive voters. We seek to investigate the effect of changes in the reversion policy on the equilibrium policy that emerges. We require one additional assumption. Let $\Delta v(\tau, y) = v(\tau, y) - v(\tau_0, y)$ denote the utility improvement for a y-type voter from policy τ over the reversion policy. We say utility improvements are log-submodular if $\frac{\partial^2}{\partial y \partial \tau} \log \Delta v(\tau, y) \leq 0$. Roughly speaking, log-submodularity of utility improvements implies that, the coefficient of absolute risk aversion (over the tax rate) of the indirect utility function $v(\tau, y)$ is increasing in agent's income.[20] [21] Equivalently, log-submodularity implies that when policy moves in a direction that improves utility, the coefficient of absolute risk aversion decreases.

Proposition 4. *Fix any decision rule d. Let $t^*(\tau_0)$ be the equilibrium tax rate, given reversion policy τ_0.*

- *If $\tau_0 \in [\tau_R, \tau_L]$, then $t^*(\tau_0) = \tau_0$, and so: $\frac{\partial t^*}{\partial \tau_0} = 1$*

- *If $\tau_0 \notin [\tau_R, \tau_L]$ and preferences exhibit log-submodular improvements, then $\frac{\partial t^*}{\partial \tau_0} < 0$*

Proposition 4, illustrated graphically in Figure 2, demonstrates the effect of a static reversion policy on the equilibrium tax rate. First, and trivially, when the reversion policy is a core policy, our refinement selects the reversion policy. Hence, changes in the reversion policy will be matched by identical changes in the equilibrium policy.

Second, if the reversion policy is not in the core, then as the reversion policy becomes more extreme, the equilibrium policy becomes more moderate. This echoes the comparative static

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[20] When utility is increasing, the coefficient of absolute risk aversion is defined by $A(\tau, y) = -\frac{v_{\tau\tau}(\tau,y)}{v_\tau(\tau,y)}$ which takes a positive value if the agent is risk averse over tax policy. The appropriate analogue when utility is decreasing is: $A(\tau, y) = \frac{v_{\tau\tau}(\tau,y)}{v_\tau(\tau,y)}$.

[21] To see this, suppose without loss of generality, that policy $\tau > \tau_0$ is an improvement over the reversion policy. By the mean value theorem, there exists $\gamma \in (\tau_0, \tau)$ such that $\Delta v(\tau, y) = (\tau - \tau_0) \cdot v_\tau(\gamma, y)$. Then $\frac{\partial}{\partial \tau} \log \Delta v(\tau, y) = \frac{v_{\tau\tau}(\gamma,y)}{v_\tau(\gamma,y)} \cdot \frac{\partial \gamma}{\partial \tau} + \frac{1}{\tau - \tau_0}$. It is easily verified that $\frac{\partial \gamma}{\partial \tau} > 0$. Hence, $\frac{\partial^2}{\partial y \partial \tau} \log \Delta v(\tau, y) < 0$ provided that $-\frac{v_{\tau\tau}(\gamma)}{v_\tau(\gamma)}$ is increasing in y.

result in Banks and Duggan (2006) that a reversion policy closer to the core will result in equilibrium policies close to the reversion policy. To build intuition for this result, suppose $\tau_0 < \tau_R$. Consider a marginal increase in τ_0, which brings the reversion policy closer to the core and thus makes it less extreme. Disagreement is now less costly for both players, although, by the log-submodularity of utility improvements, the relative utility gain is larger for the right decisive voter. Intuitively, this will increase the bargaining strength of the right decisive voter. One way to see this is to note that Nash Bargaining maximizes the (weighted) product of the decisive agents' gains over the reversion policy. Since the potential gains for the right decisive voter are now smaller relative to the potential gains for the left decisive voter, the solution must realize a larger share of the right decisive voter's potential gains. This moves the equilibrium policy closer to her ideal policy – making it more extreme, in the sense of being closer to the boundary of the core.
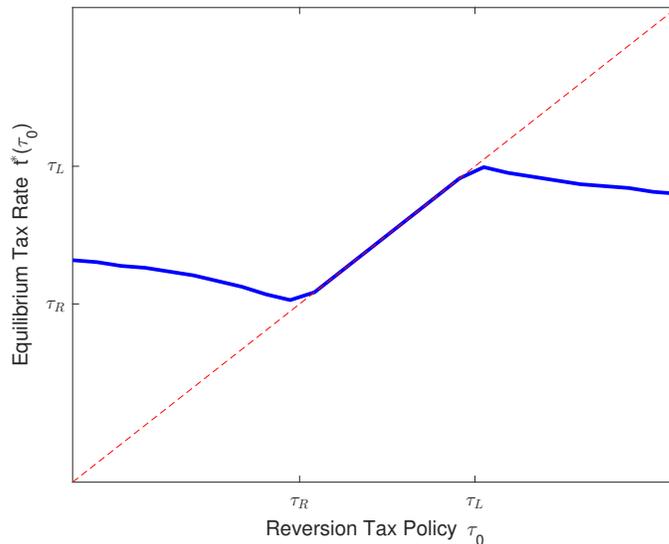

Figure 2: Effect of Changing the Reversion Policy. The income distribution is the same as in Figure 1, and decision rule requires a 55 percent super-majority.

Two features of Proposition 4 are worth mentioning. First, unlike the case of simple-majority, where the equilibrium policy is independent of the reversion policy, for general decision rules, the reversion policy significantly affects the policy that eventuates. As part 1 of the Proposition shows, the equilibrium policy is simply the reversion policy (τ_0) whenever τ_0 lies in the core. In the context of super-majority rules, this result reflects the well-known 'status-quo' bias —although the result holds more generally. However, as part 2 of the Proposition shows, the reversion policy continues to influence the equilibrium policy, even when the reversion policy lies outside the core. As a general matter, the policies that are chosen will

depend on the policies that would have otherwise resulted had we failed to choose.

Second, as Figure 2 makes clear, the mapping from reversion policies to equilibrium policies is non-monotone.[22] Moreover, if the reversion policy lies outside the core, then the polity will most likely implement moderate policies, when the reversion policy is extreme. Our analysis motivates why polities may choose extreme (and even Pareto inferior) reversion policies. The 2013 budget sequester is a particularly compelling example. More generally, bills with sunset clauses that cause policy to reset to some Pareto inferior outcome enable Congress to re-evaluate policy reforms in a way that expands the scope of policies that may be implemented in the future.

4.2   Changing the Decision Rule

We now explore the effect of changing the decision rule itself. To do so, we limit attention to super-majority rules, for two reasons. First, given recent State trends towards implementing super-majority requirements, the consequences of introducing and increasing super-majority rules are particularly salient. Second, this sub-class of decision rules is easily parameterized, and is thus particularly conducive to comparative static analysis.

In doing so, we begin by noticing that increasing the super-majority requirement expands the core to include more of both lower and higher tax policies. Although 'conventional wisdom' might be that larger super-majority requirements induce lower taxation and redistribution, and indeed this wisdom has empirical support, the theoretical basis for this wisdom is not immediately evident. Thus, our goal in this section is to explore, in greater detail, the mechanisms that generate this result.

In doing so, we again require a couple of additional assumptions. Our first assumption is analogous to the log-submodularity assumption from the previous subsection. We now assume that utility improvements are log-concave in income: i.e. $\frac{\partial^2}{\partial y^2} \log \Delta v(\tau, y) \leq 0$. By the Spence-Mirrlees condition, we know that the benefit of a tax increase is smaller (or even negative) for richer voters than poorer ones. If Δv were *concave* in y, then not only is a given tax increase less desirable for richer voters, but the utility gain is decreasing at an increasing rate. Given our intuitions about the burden of taxation, this property would seem to be rather natural, although it isn't an immediate consequence of the baseline assumptions. It is also stronger than we need. We only require that Δv is log-concave in income, which allows

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[22]This non-monotonicity is not present in the selection procedure advocated by Gradstein (1999) and Dal Bo (2006).

the marginal disutility of taxation to grow at a decreasing rate, as long as this rate is not too large.

Our second assumption concerns the distribution of income. It has long been recognized (see Romer (1975), Meltzer and Richard (1981)) that the extent of redistribution is affected by the shape of the income distribution, and in particular, its skewness. As conventionally defined, a distribution's skewness depends on the properties of its third moment. We introduce a different notion of skewness, similar to Boshnakov (2007) and Critchley and Jones (2008), that depends on the properties of the density function, directly. Our alternative notion, which we call *skewness**, is stronger than the conventional notion, but we think it reasonably applies to typical income profiles.

We say a distribution F is *right-skewed** if $f(F^{-1}(p)) < f(F^{-1}(1-p))$ for all $p \in (\frac{1}{2}, 1)$. *Left-skewness** is defined analogously, reversing the sign of the inequality. To understand the implications of our concept, suppose F is right-skewed*, and take any quantiles p', p satisfying $\frac{1}{2} \leq p' < p < 1$. Then, the difference in incomes of agents at quantiles p' and p is larger than the difference in incomes at quantiles $1-p$ to $1-p'$. Formally, $F^{-1}(p) - F^{-1}(p') \geq F^{-1}(1-p') - F^{-1}(1-p)$. Setting $p' = \frac{1}{2}$, it is immediate from right-skewness*, that for every $p > \frac{1}{2}$, the income at quantile p is farther from the median than the income at quantile $1-p$. This relationship, in ratio form, is very similar to measures of skewness advocated by Pogorelskiy and Traub (2017) and Groeneveld, Meeden et al. (2009).[23] Naturally, right-skewness* implies that the mean income lies above the median.

Although the skewness* concept is stronger than regular skewness, the property holds for commonly used families of distributions including the log-normal, Pareto, and Gamma distributions (which includes the Chi-Square distribution as a special case). The Weibull distribution (which includes the Exponential distribution as a special case) is right-skewed* provided the shape parameter κ is not too large.

For any super-majority requirement $q \geq \frac{1}{2}$, let $t^*(q)$ be the equilibrium tax rate, and $y^*(q) = \tau^{-1}(t^*(q))$ be the income of the pivotal voter. Additionally, let $y_0 = \tau^{-1}(\tau_0)$ be the income of the voter whose ideal policy coincides with the reversion policy.[24]

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[23]Pogorelskiy and Traub (2017) measure right-skewness by the ratio: $\frac{F^{-1}(p) - F^{-1}(\frac{1}{2})}{F^{-1}(\frac{1}{2}) - F^{-1}(1-p)} - 1$, for some exogenously chosen p. A distribution is right-skewed if the measure is larger than 0. Groeneveld, Meeden et al. (2009) measure skewness by taking the *sup* of the Pogorelskiy and Traub (2017) measure, over all $p \in (0.5, 1)$.

[24]If $\tau_0 = 0$, then it is ideal for every voter with income $y \geq \overline{y}$. We take the lowest income earner amongst these, i.e. $\overline{y}$.

Proposition 5. *Suppose $\tau_0 < \tau_{med}$ so that $y_0 > y_{med}$. The equilibrium tax rate has the following properties:*

- *If $q = \frac{1}{2}$, then the median voter is decisive and $t^* = \tau_{med}$.*

- *If $q \geq F(y_0)$, then $t^*(q) = \tau_0$*

- *If $q \in \left(\frac{1}{2}, F(y_0)\right)$, then $t^*(q) > \tau_0$. Furthermore, if utility improvements are log-concave in income and if F is right-skewed*, then $t^*(q)$ is strictly decreasing, and $y^*(q)$ is strictly increasing, in q.*

Proposition 5 demonstrates the effect of varying the super-majority requirement on the equilibrium level of taxation and redistribution, assuming a reversion policy below the median voter's ideal. Several features are worth noting. First, our results are consistent with Meltzer and Richard (1981) in that, under simple-majority rule, the equilibrium tax rate coincides with the ideal policy of the median voter. The median voter theorem obtains. Second, since the size of the core is increasing in the required super-majority, for q sufficiently large (i.e. for $q \geq F(y_0)$), the core will have expanded sufficiently to include the reversion policy. If so, by Proposition 1, the equilibrium policy is simply the reversion policy.

The most interesting case arises when q takes an intermediate value. If so, we know that $t^* > \tau_0$, since the Nash Bargaining solution must be contained in the core$^+$. Moreover, if utility improvements are log-concave in income and if the income distribution is right-skewed*, the equilibrium tax rate decreases monotonically from τ_{med} to τ_0 as q increases from $\frac{1}{2}$ (simple majority rule) to $F(y_0)$.

There are two effects at play that cause this result to be true. First, increasing the super-majority requirement causes both the left and right decisive voters to become more 'extreme' (in the sense that the left decisive voter becomes poorer and therefore demands even more redistribution, and the right decisive voter becomes richer and demands even less redistribution). But right-skewness* implies that, for a given increase in q, the income of the right decisive voter increases by more than the income of the left decisive voter decreases. Since Nash Bargaining, in effect, selects a 'pivotal' voter whose income is an 'average' of the incomes of the left and right decisive voters, skewness causes this average to be higher —and this pushes policy in the right decisive voter's favored direction; the equilibrium tax decreases.

Additionally, there is a second effect that would be present even if the income distribution were not skewed. Since the right decisive voter's ideal policy is now closer to the reversion

policy, disagreement is less painful for her. By contrast, the left decisive voter's ideal policy is now even further from the reversion policy, so disagreement becomes costlier. This will increase the relative bargaining strength of the right decisive voter. The intuition is analogous to the case of a changing status quo discussed in the previous sub-section. Nash Bargaining maximizes the (weighted) product of the decisive agents' gains over the status quo. Since the potential gains for the right decisive voter are now smaller, and the potential gains for left decisive voter are larger, the solution must realize a larger share of the right decisive voter's potential gains. Increasing the super-majority rule gives an in-built advantage to the player whose ideal policy is closer to the reversion policy. When the reversion policy is 'low', this pulls the equilibrium policy further towards the ideal policy of the rich.

We note that this second effect would work in the opposite direction (i.e. it would skew taxes in favor of poorer agents) if the reversion policy were 'high'. However, as long as the income distribution is right-skewed*, the first effect will continue to privilege richer agents. Hence, when the reversion policy is 'high', the overall effect of increasing the super-majority requirement is ambiguous.
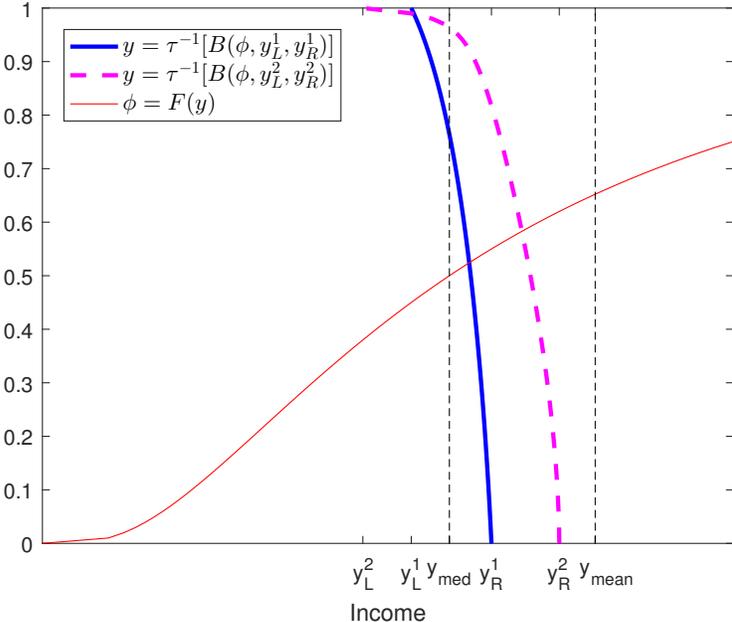
Figure 3: Effect of Changing the Required Super-Majority. The income distribution is the same as in Figure 1. The thick solid line corresponds to the pivotal voter's income for different bargaining weights when $q = 0.55$. The thick dashed line represents the case of $q = 0.6$. The reversion policy is assumed to be zero taxation and redistribution.

We see the two effects in Figure 3, which shows the effect of an increase in the super-majority

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requirement. As is evident, the income of the right decisive voter increases by more than the income of the left decisive voter decreases. This reflects the right-skewness* of the income distribution. Additionally, the downward sloping lines become more bowed, reflecting the greater bargaining power of the right decisive voter. As the super-majority requirement increases, for any bargaining weight, the implied income of the pivotal voter moves closer to the right decisive voter's income.

Proposition 5 and Figure 3 also demonstrate the effect of changing the super-majority rule on the size and composition of the equilibrium factions. As the required super-majority increases, the pivotal voter becomes richer, which implies a larger left faction and smaller right faction. This, in turn, changes the equilibrium bargaining weights. In fact, although the overall outcome favors the R faction, the bargaining weights are now more favorable to the L faction. The dynamic that causes the equilibrium tax rate to fall, also increases the bargaining power of the left faction, *ceteris paribus*, which partially counter-acts the decrease in the tax rate. Hence, a dynamic, akin to Le Chatelier's Principle, obtains.

4.3   Changing the Income Distribution

In this section, we consider the implications for redistribution of a changing income profile. In particular, we will focus on the effect of increasing inequality and skewness. We first note, however, that an agent's demand for redistribution depends not on the absolute level of her income, but her income relative to the mean (see equation (1)). Thus, a proportional increase in all incomes (which implies an improvement in the sense of first order stochastic dominance) should have no effect on the demand for redistribution. Intuitively, for a change in the income profile to affect redistribution, it must be that the distribution of relative incomes changes. When comparing two income distributions, the means of those distributions are inconsequential to the question of equilibrium redistribution —what matters is the distribution of incomes relative to the mean.

Let F and G be two income distributions that are continuously differentiable and admit densities f and g. Following Moyes (1994), we say G is *more unequal*[25] than F if:

$$\frac{F^{-1}(q)}{F^{-1}(p)} \leq \frac{G^{-1}(q)}{G^{-1}(p)} \text{ for all } 0 < p < q < 1$$

Under this definition, inequality increases if, when comparing incomes at any two different quantiles, the income of the richer agent, relative to the poorer one, becomes larger. The

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[25]Moyes (1994) writes 'F dominates G in *relative differentials*'.

definition is consistent with several common quantile-based measures of inequality, such as the 80-20, 90-10 and 90-50 ratios. Our ordering is also consistent with the Lorenz order. In fact, as Moyes (1994) establishes, our condition implies that Lorenz dominance holds over arbitrary subsets of the population, as well as the population as a whole. Finally, if G is more unequal than F under our definition, it will also have a higher associated Gini coefficient.

Our definition is also consistent with the behavior of all of the commonly used families of distributions discussed in the previous subsection (log-normal, Pareto, etc.) Each of those distributions is characterized by two parameters: a scale or location parameter, and a shape or variance parameter. For each family, a change in the shape parameter that causes the variance to increase also causes inequality to increase, whilst a change in the scale/location parameter has no effect on inequality.

It is straight-forward to show that if G is more unequal than F, then G crosses F only once and from above.[26] We say G is a *simple spread* of F. If $\overline{y_F} \geq \overline{y_G}$, then by Rothschild and Stiglitz (1970), F second-order stochastically dominates G. If $\overline{y_F} = \overline{y_G}$, G is a mean-preserving spread of F.

Recall that, what matters is the distribution of incomes relative to the mean. We show that if G is more unequal than F, then the distributions of relative incomes also have a single crossing.[27] Formally, there is a pair $(\hat{\kappa}, \hat{\rho})$ such that under both distributions F and G, the income at quantile $\hat{\rho}$ is $\hat{\kappa}$ multiples of the mean income —i.e. $\frac{F^{-1}(\hat{\rho})}{\overline{y_F}} = \hat{\kappa} = \frac{G^{-1}(\hat{\rho})}{\overline{y_G}}$. Then, by single crossing, at every quantile below $\hat{\rho}$, income (relative to the mean) is lower under G than F. Similarly, at every quantile above $\hat{\rho}$, income (relative to the mean) is higher under G than F. Income earners below quantile $\hat{\rho}$ are made relatively poorer, and income earners above are made relatively richer.

We are almost ready to state our comparative static result. To perform the comparative statics fairly, we must ensure that any effect comes from the changing income distribution, and not from coincidental changes in the decision rule or the recognition rule. To do so, we assume that the incomes of the decisive voters and the recognition probabilities are 'quantile invariant'. Formally, for $i \in \{L, R\}$, let y_i^F and y_i^G denote decisive voter i's income under distributions F and G. Additionally, let $P_F(y)$ and $P_G(y)$ denote the cumulative recognition probabilities when incomes are distributed according to F and G. Quantile invariance of decisive voters' incomes requires that $F(y_i^F) = G(y_i^G)$ for each $i \in \{L, R\}$; i.e.

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[26]I.e. there exists $\hat{y}$ s.t. $F(y) < G(y)$ for $y < \hat{y}$ and $F(y) > G(y)$ for $y > \hat{y}$.

[27]Formally, let $X \sim F$ and $Y \sim G$. Define: $\hat{X} = \frac{X}{E[X]}$ and $\hat{Y} = \frac{Y}{E[Y]}$, and let $\hat{F}$ and $\hat{G}$ denote the distributions of $\hat{X}$ and $\hat{Y}$, respectively. Then $\hat{G}$ is a mean-preserving simple-spread of $\hat{F}$.

the change in the income distribution does not change the income rank of either decisive voter. This condition is satisfied by construction whenever the decision procedure is (super)-majority rule. Quantile invariance of recognition probabilities requires that $P_F\left(F^{-1}(\rho)\right) = P_G\left(G^{-1}(\rho)\right) \equiv \Pi(\rho)$, for every ρ; the proposal power of voters at each income quantile is unchanged.

Let t_F^* and t_G^* be the equilibrium tax rates under income distributions F and G, respectively.

Proposition 6. *Suppose G is more unequal than F, that the quantile invariance properties hold, and that preferences exhibit log sub-modular improvements.*

1. *If $F(y_R^F) \leq \hat{\rho}$, then $t_G^* \geq t_F^*$, with equality only if $t_F^* = \tau_0 = t_G^*$.*

2. *If $F(y_L^F) \geq \hat{\rho}$, then $t_G^* \leq t_F^*$, with equality only if $t_F^* = \tau_0 = t_G^*$.*

3. *If $F(y_L^F) < \hat{\rho} < F(y_R^F)$, then it is ambiguous whether taxes will be higher or lower under G.*

Proposition 6 states that, roughly speaking, if both decisive voters are 'poor', then redistribution will increase when inequality increases, whilst if the decisive voters are both 'rich', then redistribution will decrease, where 'rich' and 'poor' designate income quantiles above and below the threshold $\hat{\rho}$. As in the previous subsection, there are two effects that cause this to be true. For concreteness, we describe the intuition for the first case, noting that the other case is analogous.

First, recall that increasing inequality makes the rich relatively richer and the poor relatively poorer. Since both decisive voters are 'poor', they are made poorer relative to the mean under G, and this increases their demand for redistribution. Then, since the equilibrium tax rate is a generalized mean of the ideal tax rates of the decisive voters, the equilibrium tax rate will be higher as well. Second, we saw in Section 4.1 that the effective bargaining strength of each decisive voter depends on the distances between their ideal policies and the reversion policy. Since the ideal tax rates of both decisive voters increase, then the reversion policy becomes more extreme if it is low, and more moderate if it is high. In either case, by the logic in Section 4.1, bargaining favors the left decisive voter, which causes redistribution to rise even further.

Naturally, if both decisive voters are 'rich', then both effects push in the direction of less redistribution. If the decisive voters straddle the divide between 'rich' and 'poor', then their

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demands for redistribution go in opposite directions. The net effect depends on how much each decisive voter's relative income changes, and the location of the reversion policy (which will skew bargaining power in favor of one decisive voter over the other).

The location of the threshold quantile $\hat{\rho}$ is crucial in determining the qualitative implications of increasing inequality. The fact of increasing inequality itself places no restriction on the location of $\hat{\rho}$. For example, $\hat{\rho} \gtrless \frac{1}{2}$, and so increasing inequality has ambiguous effect on the median income relative to the mean. An implication is that, contra Meltzer and Richard (1981), even in the case of simple majority rule, an increase in inequality is not, by itself, sufficient to generate higher taxation and redistribution.

To say more, we need additional structure. Similar to our definition of 'more unequal', we say G is more right-skewed* than F if $\frac{g(G^{-1}(p))}{g(G^{-1}(1-p))} < \frac{f(F^{-1}(p))}{f(F^{-1}(1-p))}$ for every $p \in (0.5, 1)$. We show in the proof of Lemma 1, that if G is more right-skewed* than F, then $G(\overline{y_G}) > F(\overline{y_F})$ —the average income moves to a higher quantile. Thus, our notion of more-skewed* has implications analogous to other measures of skewness that compare the (standardized) distance between the mean and median, such as Pearson's second skewness coefficient. Combined with the single-crossing property of increasing inequality, the fact that the mean income rises to higher quantile implies that $\hat{\kappa} > 1$ or that $\hat{\rho} > G(\overline{y_G}) > F(\overline{y_F})$.

The notions of inequality and skewness are distinct, and, in principle, it is possible for a change in the income distribution to increase inequality but decrease skewness, or vice versa.[28] However, for all of the commonly used families of distributions, a change in the shape parameter that causes inequality to increase also causes the income distribution to become more right-skewed*. This is consistent with recent empirical evidence indicating that income inequality in developed democracies has been increasing primarily as a result of faster increases in top income shares relative to bottom ones (see Piketty and Saez (2003), and Guvenen and Kaplan (2017)). Moreover, skewness has been historically argued as a means of measuring income inequality. of income distributions (see Bowman (1945), and Alker Jr and Russett (1964)). More recently, Pogorelskiy and Traub (2017) utilize a slightly weaker notion of skewness than our definition to characterize increasing inequality and demand for redistribution in the United Kingdom.

For the results that follow, we assume that preferences exhibit log-submodular improvements, and that the quantile invariance properties are satisfied.

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[28]For example, consider the class of distributions on $[0, 1]$ having linear density. Let F be the uniform distribution on $[0, 1]$ and G be the distribution with density $g(y) = 2y$. Then G is more right-skewed* than F, but F is more unequal than G. See also Cowell (2011).

Lemma 1. *Suppose G is more unequal and more right-skewed* than F. If both decisive voters under F both have below mean income, then $t_G^* \geq t_F^*$.*

Recall, any agent with below average income prefers strictly positive redistribution. When the income distribution is right-skewed*, the mean income lies above the median income, and so the majority of agents will favor redistribution. In fact, as income becomes more right-skewed*, the gap between mean and median increases, and so the subset of the population favoring positive redistribution grows. Lemma 1 shows that, so long as both decisive voters originally favored positive redistribution (i.e. had below average income), then an inequality and skewness increasing change in the income distribution will cause the equilibrium demand for redistribution to increase. (Such a change will also increase the likelihood that the decisive voters have below average incomes.) In the special case of (super)-majority rules, given the discussion in the previous subsection, we know that the antecedent condition will be satisfied as long as the super-majority requirement is not too demanding. Formally:

Corollary 1. *Suppose G is more unequal and more right-skewed* than F, and that the super-majority requirement is not too demanding ($q < F(\overline{y_F})$). Then $t_G^* \geq t_F^*$, with equality only if $t_F^* = \tau_0 = t_G^*$.*

Figure 4 illustrates the insights from Corollary 1. Income distribution G is more unequal and more right-skewed* than F. (In fact, both distributions have a log-normal distribution with the same mean, but different variances.) As the diagram shows, G crosses F once, and from above. The crossing occurs at quantile $\hat{\rho}$ and at income level $\hat{k}\overline{y}$, where $\hat{k} > 1$. This implies that all agents with below-mean income and even those with slightly above-mean incomes are made relatively poorer when the income distribution becomes more unequal. By contrast, the very rich are made relatively richer. Since both decisive voters have below-mean incomes, their demand for redistribution increases. The pivotal voter becomes poorer, and so equilibrium redistribution increases.

Our results presume that increasing inequality doesn't, in itself, change the distribution of political power within the electorate. We think this is the appropriate way to do comparative statics —holding all other factors constant. However, we acknowledge that, in reality, it may well be that rising inequality is accompanied by a greater concentration of political power within the hands of the rich as suggested in the literature on elite capture. In our model, this could be captured both by an effective change in the decision rule that causes at least one of the decisive voters to become richer, or a change in the recognition rule that gives more agenda control to the rich. Such contemporaneous changes in the decision or recognition
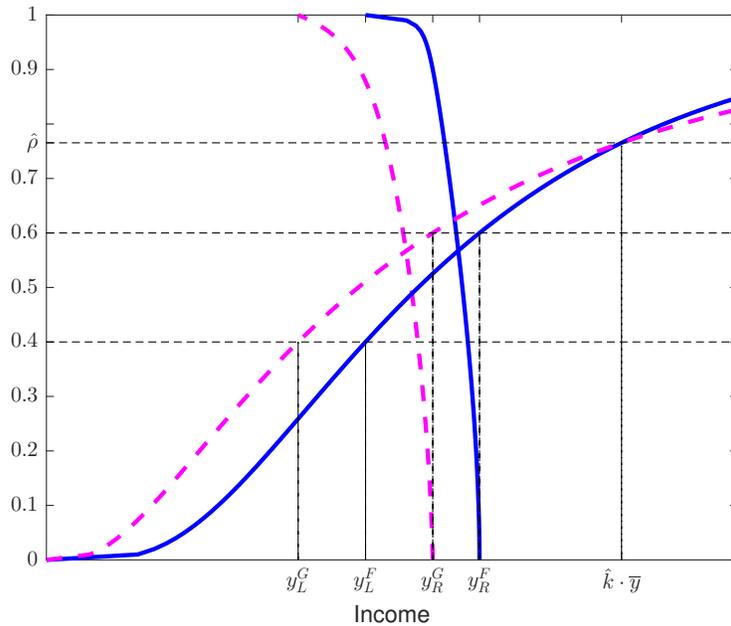
Figure 4: Effect of Changing the Income Distribution. The decision procedure is 60% super-majority rule. The income distributions are log-normally distributed with identical means. The solid lines correspond to the income distribution and bargaining solutions when the Gini coefficient is 0.33. The thick dashed lines correspond to a Gini coefficient of 0.45. The reversion policy is $\tau_0 = 0$.

rules would have the natural effect of reducing the equilibrium level of redistribution, *ceteris paribus*.

5   Conclusion

The median voter theorem is ubiquitous in models of democratic decision-making. However, in many contexts, polities and committees deliberate under alternative decision procedures where the median voter theorem does not apply. A difficulty with considering such procedures is that, generally, they admit multiple core policies, which makes equilibrium prediction complicated.

This paper suggests a refinement that selects a unique robust policy from within the core for any arbitrary decision procedure. Our selection is motivated by the observation that when delay is costless, the equilibria of bargaining under the canonical protocol of Baron and Ferejohn (1989) exactly coincide with the set of policies in the core[+]. By contrast,

introducing even small costs to delay results in the bargaining game admitting a unique equilibrium. Taking the limit as costs become negligible, then, selects the unique policy from within the core that is robust to introducing small costs to making counter-proposals.

Our refinement has a simple and elegant characterization – it selects the asymmetric Nash Bargaining solution in a bargain between the left and right decisive voters (whose identities are determined by the decision rule), with endogenous bargaining weights that depend on the full income profile. Our analysis suggests a generalized version of Duverger's Law – that regardless of the decision procedure, heterogeneous voters will likely separate into two cohesive factions. However, unlike the simple-majority case, these factions need not be equally sized, and may be represented by non-median factional leaders. For example, under super-majority rule, our refinement predicts a larger faction of the poor, and a smaller faction of the rich.

In contrast to median voter logic, which always selects the median voter's ideal policy, we show that for arbitrary decision rules, the equilibrium policy will generically depend on the location of the reversion policy. Moreover, we show that this mapping is non-monotone, and that the equilibrium policy becomes more moderate as the reversion policy becomes more extreme.

We apply our proposed refinement to analyze the effect on taxation and redistribution of introducing or increasing super-majority requirements. After such a change, the core expands to include both higher and lower tax policies, so that, absent a consistent selection procedure, the implications of changing the super-majority requirement are unclear. We show that, under standard assumptions, the equilibrium tax rate is decreasing in the size of the required super-majority. Further, we decompose this result into two effects that are complementary when the consequence of disagreement is for redistribution to decrease, but which have opposing effects when the reversion tax policy is 'high'. Thus, we provide a theoretical basis for both the 'common wisdom' and empirical evidence that super-majority rules result in less redistribution, although we demonstrate that this result is sensitive to the properties of the reversion policy.

Additionally, we join the conversation on the relationship between redistribution and the degree of inequality in a polity. Importantly, we show that this relationship depends on the nature of the decision rule being applied. If both decisive voters are 'poor', then redistribution will rise, whilst if at least one decisive voter is sufficiently 'rich', then redistribution will fall. The threshold that delineates 'rich' and 'poor' itself depends on the properties of the income distribution. If increasing inequality is accompanied by increasing skewness,

then this threshold lies above the mean (and thus median) income. Thus, our results are consistent with Meltzer and Richard (1981), who show that increasing inequality, in the form of increasing skewness, causes higher redistribution under simple majority rule. However, our results also admit the possibility of lower redistribution if at least one decisive voter has well-above average income —i.e. in the event of elite capture. Our model, therefore, suggests mechanisms that may reconcile the inconsistent empirical evidence on the effect on redistribution of increasing inequality.

Appendices

A   Proofs

Proof of Proposition 1. The proof follows the strategy in Cardona and Ponsati (2011). Suppose the equilibrium social acceptance set is $[\underline{t}, \overline{t}]$. It is immediate that the equilibrium proposals are given by:

$$
t(y) = \begin{cases} \underline{t} & y > \tau^{-1}(\underline{t}) \\ \tau(y) & y \in \left[\tau^{-1}(\underline{t}), \tau^{-1}(\overline{t})\right] \\ \overline{t} & y < \tau^{-1}(\overline{t}) \end{cases}
$$

Let $V(\underline{t}, \overline{t}; y) = (1 - P(\tau^{-1}(\underline{t})))v(\underline{t}, y) + \int_{\tau^{-1}(\overline{t})}^{\tau^{-1}(\underline{t})} v(\tau(z), y)dP(z) + P(\tau^{-1}(\overline{t}))v(\overline{t}, y)$ be the expected utility of a type-y agent in the continuation game. Let the individual acceptance sets $A(y) = [\underline{t}(y), \overline{t}(y)]$ be defined as in the statement of the proposition.

Step 1. We first show that in any equilibrium, $\underline{t} = \underline{t}(y_L)$ and $\overline{t} = \overline{t}(y_R)$. Take any y, y' with $y' < y$. The following claims are true:

1. Suppose $v(\underline{t}, y) \leq (1-\delta)v(\tau_0, y) + \delta V(\underline{t}, \overline{t}; y)$. Then $v(\underline{t}, y') \leq (1-\delta)v(\tau_0, y') + \delta V(\underline{t}, \overline{t}; y')$.

2. Suppose $v(\overline{t}, y') \leq (1-\delta)v(\tau_0, y') + \delta V(\underline{t}, \overline{t}; y')$. Then $v(\overline{t}, y) \leq (1-\delta)v(\tau_0, y) + \delta V(\underline{t}, \overline{t}; y)$.

We prove (1), and note that (2) is proved analogously. For notational simplicity, define $\Delta v(\tau, y) = v(\tau, y) - v(\tau_0, y)$ and $\Delta V(\underline{t}, \overline{t}; y) = V(\underline{t}, \overline{t}; y) - v(\tau_0, y)$. Suppose (1) is not true.

Then there exists some $y' < y$ s.t. $\Delta v(\underline{t}, y) \le \delta \Delta V(\underline{t}, \overline{t}; y)$ and $\Delta v(\underline{t}, y') > \delta \Delta V(\underline{t}, \overline{t}; y')$. This implies:

$$\Delta v(\underline{t}, y) - \Delta v(\underline{t}, y') < \delta \left[\Delta V(\underline{t}, \overline{t}; y) - \Delta V(\underline{t}, \overline{t}; y') \right]$$

Now, by the Spence-Mirlees condition, $\frac{\partial}{\partial \tau} \left[\Delta v(\tau, y) - \Delta v(\tau, y') \right] = \int_{y'}^y v_{\tau y}(\tau, z) dz \le 0$. Hence:

$$\Delta V(\underline{t}, \overline{t}; y) - \Delta V(\underline{t}, \overline{t}; y') = (1 - P(\tau^{-1}(\underline{t}))) \left[\Delta v(\underline{t}, y) - \Delta v(\underline{t}, y') \right] +$$
$$\int_{\tau^{-1}(\overline{t})}^{\tau^{-1}(\underline{t})} \left[\Delta v(\tau(z), y) - \Delta v(\tau z, y') \right] dP(z) + P(\tau^{-1}(\overline{t})) \left[\Delta v(\overline{t}, y) - \Delta v(\overline{t}, y') \right]$$
$$\le \Delta v(\underline{t}, y) - \Delta v(\underline{t}, y')$$

Then $\Delta v(\underline{t}, y) - \Delta v(\underline{t}, y') < \delta \left[\Delta V(\underline{t}, \overline{t}; y) - V(\underline{t}, \overline{t}; y') \right] \le \delta \left[\Delta v(\underline{t}, y) - \Delta v(\underline{t}, y') \right]$, which cannot be, since $\delta < 1$. Hence, the claim is true.

Let $C(t)$ be the coalition that supports policy t. Suppose $\underline{t} < \underline{t}_L$. Then, $\Delta v(\underline{t}, y_L) < \delta \Delta V(\underline{t}, \overline{t}, y_L)$, and so, by (1), $\Delta v(\underline{t}, y) < \delta \Delta V(\underline{t}, \overline{t}, y)$ for all $y < y_L$. But then, $\inf\{y \mid y \in C(t)\} > y_L$, and so, by construction, $C(t) \notin \mathcal{C}(d)$. Hence t is not socially acceptable, and so $\underline{t} \ge \underline{t}(y_L)$. Next, since $\Delta v(\underline{t}(y_L), y_L) \ge \delta \Delta V(\underline{t}, \overline{t}, y_L)$, taking the contrapositive of (1), $\Delta v(\underline{t}(y_L), y) > \delta \Delta V(\underline{t}, \overline{t}, y)$ for all $y > y_L$. This implies that there is a coalition that would accept $\underline{t}(y_L)$, and so $\underline{t} \le \underline{t}(y_L)$. Hence $\underline{t} = \underline{t}(y_L)$. By a similar argument, we can show that $\overline{t} = \overline{t}(y_R)$.

Step 2. Next, we show that the bargaining game admits a unique equilibrium. Let $\overline{\zeta}_R(\theta) = \max \{\tau \in [0, 1] \mid \Delta v(\tau, y_R) \ge \delta \Delta V(\theta, \tau; y_R)\}$ and $\underline{\zeta}_L(\theta) = \min \{\tau \in [0, 1] \mid \Delta v(\tau, y_L) \ge \delta \Delta V(\tau, \theta; y_L)\}$. Naturally, if $(\underline{t}, \overline{t})$ are a pair of equilibrium thresholds, we must have: $\underline{t} = \underline{\zeta}_L(\overline{t})$ and $\overline{t} = \overline{\zeta}_R(\underline{t})$. Let $H(\underline{t}) = \underline{\zeta}_L (\overline{\zeta}_R(\underline{t}))$. Then $(\underline{t}, \overline{t})$ is an equilibrium if $\underline{t}$ is a fixed point of H and $\overline{t} = \overline{\zeta}_R(\underline{t})$. Since Δv is continuous, so are $\overline{\zeta}_R$ and $\underline{\zeta}_L$. Hence, by Brouwer's fixed point theorem, H admits a fixed point.

We need to show that this fixed point is unique. Implicitly differentiating the function that defines $\overline{\zeta}_R(\theta)$, we have:

$$\overline{\zeta}_R'(\theta) = \frac{\delta(1 - P(\tau^{-1}(\theta))}{1 - \delta P(\tau^{-1}(\overline{\zeta}_R(\theta)))} \cdot \frac{v_\tau(\theta, y_R)}{v_\tau(\overline{\zeta}_R, y_R)}$$

Similarly, we have:

$$\zeta_L'(\theta) = \begin{cases} \dfrac{\delta P\left(\tau^{-1}(\theta)\right)}{1-\delta+\delta P\left(\tau^{-1}(\zeta_L(\theta))\right)} \cdot \dfrac{v_\tau(\theta,y_L)}{v_\tau(\zeta_L,y_L)} & \zeta_L(\theta) > 0 \\ 0 & \zeta_L(\theta) = 0 \end{cases}$$

Let $(\underline{t},\bar{t})$ be equilibrium thresholds (which implies $\overline{\zeta}_R(\underline{t}) = \bar{t}$ and $\underline{\zeta}_L(\bar{t}) = \underline{t}$). Then:

$$H'(\underline{t}) = \begin{cases} \dfrac{\delta\left(1-P\left(\tau^{-1}(\underline{t})\right)\right)}{1-\delta P\left(\tau^{-1}(\bar{t})\right)} \cdot \dfrac{\delta P\left(\tau^{-1}(\bar{t})\right)}{1-\delta+\delta P(\tau^{-1}(\underline{t}))} \cdot \dfrac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)} \cdot \dfrac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)} & \underline{t} > 0 \\ 0 & \underline{t} = 0 \end{cases}$$

We seek to show that $H'(\underline{t}) < 1$ at any fixed point $\underline{t}$. Notice that this is immediate if $\underline{t} = 0$. Suppose $\underline{t} > 0$. Then $H'(\underline{t})$ is the product of 4 terms, the first two of which are positive and less than 1. It suffices then to show that the product of the third and fourth terms is also less than 1.

Suppose $H(\underline{x}) \geq 1$. Then at least one of $\left|\dfrac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| > 1$ or $\left|\dfrac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)}\right| > 1$. There are several cases to consider. First, suppose $\left|\dfrac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| > 1$. Since $\bar{t} > \tau(y_R)$ then $v_\tau(\bar{t},y_R) < 0$ by the concavity of v. If $\tau(y_R) \leq \underline{t} < \bar{t}$, then concavity implies $v_\tau(\bar{t}) < v_\tau(\underline{t}) \leq 0$, which contradicts $\left|\dfrac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| > 1$. Hence $\underline{t} < \tau(y_R) < \bar{t}$, and so $v_\tau(\underline{t}) > 0$. Suppose additionally $\tau(y_L) \geq \bar{t} > \underline{t}$. Then $v_\tau(\underline{t},y_L) > 0$ and $v_\tau(\bar{t},y_L) \geq 0$. Hence $\dfrac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)} < 0$, and $\dfrac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)} > 0$, and so $H < 0$, which cannot be. Hence $\underline{t} < \tau(y_R) < \tau(y_L) < \bar{t}$. Then, by the Spence-Mirrlees condition, $0 < v_\tau(\underline{t},y_R) < v_\tau(\underline{t},y_L)$ and $v_\tau(\bar{t},y_R) < v_\tau(\bar{t},y_L) < 0$, and so:

$$\frac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)} \cdot \frac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)} = \frac{v_\tau(\underline{t},y_R)}{v_\tau(\underline{t},y_L)} \cdot \frac{v_\tau(\bar{t},y_L)}{v_\tau(\bar{t},y_R)} < 1$$

Hence $H < 1$, which cannot be, and so $\left|\dfrac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| \leq 1$.

By a similar logic, we show that $\left|\dfrac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)}\right| \leq 1$. Hence our initial supposition was wrong; $H'(\underline{t}) \not\geq 1$. Hence, $H' < 1$ and so H admits a unique fixed point. $\qquad\square$

Proof of Proposition 2. Recall, the acceptance set is $[\underline{t},\bar{t}]$, where $\Delta v(\underline{t},y_L) = \delta\Delta V(\underline{t},\bar{t},y_L)$ and $\Delta v(\bar{t},y_R) = \delta\Delta V(\underline{t},\bar{t},y_R)$. Now, by construction, $\Delta v(\bar{t},y_L) \geq \delta\Delta V(\underline{t},\bar{t},y_L)$ since y_L will accept $\bar{t}$. Then, since $\Delta v(\tau,y_R)$ is strictly quasi-concave in τ for each y, $\Delta v(t,y_L) > \Delta v(\underline{t},y_L)$ for every $t \in (\underline{t},\bar{t})$. Similarly, $\Delta v(\bar{t},y_R) \geq \delta\Delta V(\underline{t},\bar{t},y_R)$ and so $\Delta v(t,y_R) > \Delta V(\bar{t},y_R)$ for every $t \in (\underline{t},\bar{t})$. Hence $\Delta V(\underline{t},\bar{t},y_L) > \Delta v(\underline{t},y_L)$ and $\Delta V(\underline{t},\bar{t},y_R) > \Delta v(\bar{t},y_R)$ whenever $\underline{t} < \bar{t}$. Now, for every $\delta < 1$, $\dfrac{\Delta v(\underline{t},y_L)}{\Delta V(\underline{t},\bar{t},y_L)} = \delta = \dfrac{\Delta v(\bar{t},y_R)}{\Delta V(\underline{t},\bar{t},x_R)}$ and so as $\delta \to 1$,

we need $\Delta V(\underline{t}, \bar{t}, y_L) - \Delta v(\underline{t}, y_L) \to 0$ and $\Delta V(\underline{t}, \bar{t}, y_R) - \Delta v(\bar{t}, y_R) \to 0$. But this requires $\bar{t} - \underline{t} \to 0$. Hence $A \to [t^*, t^*]$ as $\delta \to 1$. $\qquad\square$

Proof of Proposition 3. For notational convenience, denote: $\Delta v_i(\tau) = v(\tau, y_i)$ for $i \in \{L, R\}$, and denote $\underline{t}(y_L) = \underline{t}_L$ and $\bar{t}(y_R) = \bar{t}_R$. For every $\delta < 1$, we know that $\underline{t}_L$ and $\bar{t}_R$ are defined by the system:

$$\Delta v_R\left(\bar{t}_R\right) = \delta \left[F\left(\tau^{-1}(\bar{t}_R)\right) \Delta v_R(\bar{t}_R) + \int_{\tau^{-1}(\bar{t}_R)}^{\tau^{-1}(\underline{t}_L)} \Delta v_R\left(\tau(z)\right) dF(z) + \left(1 - F\left(\tau^{-1}(\underline{t}_L)\right)\right) \Delta v_R(\underline{t}_L) \right]$$

$$\Delta v_L\left(\underline{t}_L\right) = \delta \left[F\left(\tau^{-1}(\bar{t}_R)\right) \Delta v_L(\bar{t}_R) + \int_{\tau^{-1}(\bar{t}_R)}^{\tau^{-1}(\underline{t}_L)} \Delta v_L\left(\tau(z)\right) dF(z) + \left(1 - F\left(\tau^{-1}(\underline{t}_L)\right)\right) \Delta v_L(\underline{t}_L) \right]$$

Fix some $\varphi \in [0, 1]$. Let $E = (1 - \varphi)\bar{t}_R + \varphi \underline{t}_L$, and let $\varepsilon = E - \underline{t}_L$. Note that $(\bar{t}_R, \underline{t}_L)$ uniquely pins down (E, ε), and that these are implicitly functions of δ. (Indeed, $\underline{t}_L = E - \varepsilon$ and $\bar{t}_R = E + \frac{\varphi}{1-\varphi}\varepsilon$, and $\varepsilon \to 0$ as $\delta \to 1$.) Affect this change of variables. We have:

$$\left[1 - \delta P\left(\tau^{-1}\left(E + \frac{\varphi}{1-\varphi}\varepsilon\right)\right)\right] \Delta v_R\left(E + \frac{\varphi}{1-\varphi}\varepsilon\right) = \delta \int_{\tau^{-1}\left(E + \frac{\varphi}{1-\varphi}\varepsilon\right)}^{\tau^{-1}(E-\varepsilon)} \Delta v_R\left(\tau(z)\right) dP(z) +$$
$$+ \delta \left(1 - P\left(\tau^{-1}(E - \varepsilon)\right)\right) \Delta v_R(E - \varepsilon) \tag{3}$$

$$\left[1 - \delta\left(1 - P\left(\tau^{-1}(E - \varepsilon)\right)\right)\right] \Delta v_L(E - \varepsilon) = \delta \int_{\tau^{-1}(E+\frac{\varphi}{1-\varphi}\varepsilon)}^{\tau^{-1}(E-\varepsilon)} \Delta v_L\left(\tau(z)\right) dP(z) +$$
$$+ \delta P\left(\tau^{-1}(E + \frac{\varphi}{1-\varphi}\varepsilon)\right) \Delta v_L\left(E + \frac{\varphi}{1-\varphi}\varepsilon\right) \tag{4}$$

Totally differentiating (3) w.r.t δ gives:

$$\left\{ \left[1 - \delta P\left(\tau^{-1}\left(\bar{t}_R\right)\right)\right] \frac{\partial \Delta v_R\left(\bar{t}_R\right)}{\partial \tau} - \delta\left(1 - P\left(\tau^{-1}\left(\underline{t}_L\right)\right)\right) \frac{\partial \Delta v_R\left(\underline{t}_L\right)}{\partial \tau} \right\} \frac{\partial E}{\partial \delta}$$
$$= P\left(\tau^{-1}\left(\bar{t}_R\right)\right) \Delta v_R\left(\bar{t}_R\right) \tag{5}$$
$$+ \left\{ \frac{\varphi}{1-\varphi} \left[1 - \delta P\left(\tau^{-1}\left(\bar{t}_R\right)\right)\right] \frac{\partial \Delta v_R\left(\bar{t}_R\right)}{\partial \tau} + \delta\left(1 - P\left(\tau^{-1}\left(\underline{t}_L\right)\right)\right) \frac{\partial \Delta v_R\left(\underline{t}_L\right)}{\partial \tau} \right\} \frac{\partial \varepsilon}{\partial \delta}$$
$$+ \int_{\tau^{-1}(\bar{t}_R)}^{\tau^{-1}(\underline{t}_L)} \Delta v_R\left(\tau(z)\right) dP(z) + \left(1 - P\left(\tau^{-1}\left(\underline{t}_L\right)\right)\right) \Delta v_R\left(\underline{t}_L\right)$$

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Taking $\delta \to 1$ gives:

$$\Delta v_R\left(t^*\right) = \frac{1}{1-\varphi}\left(1 - P\left(\tau^{-1}\left(t^*\right)\right)\right)\frac{\partial \Delta v_R\left(t^*\right)}{\partial \tau}\lim_{\delta \to 1}\frac{\partial \varepsilon}{\partial \delta}$$

which implies:

$$\frac{1}{\lim_{\delta \to 1}\frac{\partial \varepsilon}{\partial \delta}} = \frac{1}{1-\varphi}\left(1 - P\left(\tau^{-1}\left(t^*\right)\right)\right)\frac{\frac{\partial \Delta v_R\left(t^*\right)}{\partial \tau}}{\Delta v_R\left(t^*\right)}$$

Similarly differentiating (4) w.r.t. δ, and taking the limit as $\delta \to 1$ gives:

$$\frac{1}{\lim_{\delta \to 1}\frac{\partial \varepsilon}{\partial \delta}} = \frac{1}{1-\varphi}P\left(\tau^{-1}\left(t^*\right)\right)\frac{\frac{\partial \Delta v_L\left(t^*\right)}{\partial \tau}}{\Delta v_L\left(t^*\right)}$$

It follows that:

$$P\left(\tau^{-1}\left(t^*\right)\right)\frac{\frac{\partial \Delta v_L\left(t^*\right)}{\partial \tau}}{\Delta v_L\left(t^*\right)} + \left(1 - P\left(\tau^{-1}\left(t^*\right)\right)\right)\frac{\frac{\partial \Delta v_R\left(t^*\right)}{\partial \tau}}{\Delta v_R\left(t^*\right)} = 0$$

But this is precisely the Nash Bargaining solution when $\phi = P\left(\tau^{-1}\left(t^*\right)\right)$. $\qquad\square$

Proof of Proposition 4. Fix an arbitrary decision rule d. Let y_L and y_R be the implied decisive voters, and τ_L and τ_R be the associated ideal policies. Suppose $\tau_0 \notin [\tau_R, \tau_L]$. We seek to show that $\frac{\partial t^*}{\partial \tau_0} < 0$. First, let $\Delta v(\tau, y; \tau_0) = v(\tau, y) - v(\tau_0, y)$. Let ψ be dfined by:

$$\psi(\tau; \tau_0, \phi) = \phi\frac{v_\tau(\tau, y_L)}{\Delta v(\tau, y_L; \tau_0)} + (1-\phi)\frac{v_\tau(\tau, y_R)}{\Delta v(\tau, y_R; \tau_0)}$$

By the first and second order conditions, we know that $\psi(t^*(\tau_0); \tau_0, \phi^*(\tau_0)) = 0$ and that $\psi_\tau(t^*; \tau_0, \phi^*) < 0$. Additionally, since $\phi^* = P\left(\tau^{-1}(t^*)\right)$, we know that $\frac{\partial \phi^*}{\partial \tau_0} = \frac{p\left(\tau^{-1}(t^*)\right)}{\tau'(t^*)}\frac{\partial t^*}{\partial \tau_0}$. Totally differentiating $\psi(t^*(\tau_0); \tau_0, \phi^*(\tau_0)) = 0$ with respect to τ_0 gives:

$$\psi_{\tau_0}(t^*; \tau_0, \phi^*) + \left[\psi_\tau(t^*; \tau_0, \phi^*) + \psi_\phi(t^*; \tau_0, \phi^*)\frac{p\left(\tau^{-1}(t^*)\right)}{\tau'(t^*)}\right]\frac{\partial t^*}{\partial \tau_0} = 0$$

It follows that:

$$\frac{\partial t^*}{\partial \tau_0} = -\frac{\psi_{\tau_0}}{\psi_\tau + \psi_\phi\frac{p(\tau^{-1}(t^*))}{\tau'(t^*)}}$$

Now $\psi_\tau < 0$, $\psi_\phi > 0$ (since $v_\tau^L > 0$ and $v_\tau^R < 0$) and $\tau' < 0$, and so the denominator must be

negative. Hence, $\frac{\partial t*}{\partial \tau_0}$ has the same sign as ψ_{τ_0}, where:

$$\psi_{\tau_0} = \phi \frac{v_\tau(\tau, y_L)}{\Delta v(\tau, y_L; \tau_0)} \cdot \frac{v_\tau(\tau_0, y_L)}{\Delta v(\tau, y_L; \tau_0)} + (1 - \phi) \frac{v_\tau(\tau, y_R)}{\Delta v(\tau, y_R; \tau_0)} \cdot \frac{v_\tau(\tau_0, y_R)}{\Delta v(\tau, y_R; \tau_0)}$$

Furthermore, since

$$(1 - \phi) \frac{v_{\tau_0}(t^*, y_R)}{\Delta v(t^*; \tau_0, y_R)} = -\phi \frac{v_{\tau_0}(t^*, y_L)}{\Delta v(t^*; \tau_0, y_L)}$$

we have:

$$\psi_{\tau_0}(t^*; \tau_0, \phi^*) = \phi^* \frac{v_\tau(t^*, y_L)}{\Delta v(t^*; \tau_0, y_L)} \left[\frac{v_\tau(\tau_0, y_R)}{\Delta v(t^*; \tau_0, y_R)} - \frac{v_\tau(\tau_0, y_L)}{\Delta v(t^*; \tau_0, y_L)} \right] < 0$$

, where the inequality follows from the fact that Δu is log-submodular and $y_R > y_L$. $\qquad \square$

Proof of Proposition 5. Let $(t^*(q), \phi^*(q))$ be the equilibrium policy under super-majority rule q, i.e.

$$t^*(q) = \arg \max_{t \in [0,1]} [\Delta v(t, y_L(q))]^{\phi^*(q)} [\Delta v(t, y_R(q))]^{1-\phi^*(q)}$$

and $\phi^* = P(\tau^{-1}(t^*))$, where $y_L(q) = F^{-1}(1 - q)$, and $y_R(q) = F^{-1}(q)$.

Let us first dispose of the boundary cases. First suppose $q = \frac{1}{2}$. Then $y_L(q) = y_R(q) = y_{med}$, and so $B(\phi, y_L, y_R) = \tau(y_{med})$ for all $\phi \in [0, 1]$. Hence $t^*\left(\frac{1}{2}\right) = \tau(y_{med})$. Second, suppose $q \geq (y_0)$, so that $y_R \geq y_0$. Then $y_L < y_0 < y_R$. Notice that it is a necessary condition that $\Delta v(t^*, y_i) \geq 0$ for each $i \in \{L, R\}$. Since $y_L < y_0$, then $\Delta v(\tau, y_L) < 0$ whenever $\tau < \tau_0$. Similarly, since $y_R \geq y_0$, then $\Delta v(\tau, y_R) < 0$ for all $\tau > \tau_0$. Hence, for any $\phi \in [0, 1]$, the only possible solution is $B(\phi, y_L, y_R) = \tau_0$. And so $t^*(q) = \tau_0$ for all $q \geq (y_0)$.

We now consider the most interesting case. Suppose $q \in \left(\frac{1}{2}, F(y_0)\right)$. Then $\tau_L > \tau_R > \tau_0$, which implies that there is a range of $t > \tau_R$ for which $\Delta v(t, y_R) > 0$. Hence, the optimizer t^* is interior, and so satisfies the first and second order conditions. Let $\psi(\tau, \phi, q) = \phi \frac{v_\tau(t, y_L)}{\Delta v(t, y_L)} + (1 - \phi) \frac{v_\tau(t, y_R)}{\Delta v(t, y_R)}$, where $y_L = F^{-1}(1 - q)$ and $y_R = F^{-1}(q)$. Then, $\psi(t^*(q), \phi^*(q), q) = 0$ for every $q \in \left(\frac{1}{2}, F(y_0)\right)$, and $\psi_\tau(t^*, \phi^*, q) < 0$, by the second order conditions.

In what follows, we simplify notation by denoting $\Delta v^i = \Delta v(t^*, y_i)$. Take $q \in \left(\frac{1}{2}, F(y_0)\right)$, and suppose that $\frac{\partial \phi^*}{\partial q} \leq 0$. Since $\phi^* = P(\tau^{-1}(t^*))$, we know that $\frac{\partial \phi^*}{\partial q} = \frac{p(\tau^{-1}(t^*))}{\tau'(t^*)} \frac{\partial t^*}{q}$. We know that $\tau'(t) < 0$ for every t, since higher taxes are preferred by lower income earners. Hence $\frac{\partial \phi^*}{\partial q} \cdot \frac{\partial t^*}{\partial q} < 0$, and so our assumption implies $\frac{\partial t^*}{\partial q} > 0$.

Totally differentiating ψ w.r.t. q gives:

$$\psi_\tau(t^*, \phi^*, q)\frac{\partial t^*}{\partial q} + \psi_\phi \frac{\partial \phi^*}{\partial q} + \psi_q = 0$$

and so:

$$\frac{\partial t^*}{\partial q} = -\frac{\psi_\phi\left(t^*, \phi^*, q\right) \cdot \frac{\partial \phi^*}{\partial q} + \psi_q\left(t^*, \phi^*, q\right)}{\psi_t\left(t^*, \phi^*, q\right)}$$

Now, as we showed in the proof of Proposition 4, $\psi_\phi = \frac{v_\tau(t,y_L)}{\Delta v(t,y_L)} - \frac{v_\tau(t,y_R)}{\Delta v(t,y_R)} > 0$. Furthermore, as will show below, $\psi_q < 0$. Then, since $\frac{\partial \phi^*}{\partial q} \leq 0$, we have $\frac{\partial t^*}{\partial q} < 0$. But we had previously shown that $\frac{\partial t^*}{\partial q} > 0$, which is a contradiction. Hence $\frac{\partial \phi^*}{\partial q} > 0$ and $\frac{\partial t^*}{\partial q} < 0$.

It remains to show that $\psi_q < 0$. We have:

$$\psi_q = \frac{(1-\phi^*)}{f(y_R)} \cdot \frac{\Delta v^R \cdot v_{\tau y}^R - \Delta v_y^R \cdot v_\tau^R}{(\Delta v^R)^2} - \frac{\phi^*}{f(Y_L)} \cdot \frac{\Delta v^L \cdot v_{\tau y}^L - \Delta v_y^L \cdot v_\tau^L}{(\Delta v^L)^2}$$

$$= \frac{(1-\phi^*)}{f(y_R)} \frac{v_\tau^R}{\Delta v^R} \left[\frac{v_{\tau y}^R}{v_\tau^R} - \frac{\Delta v_y^R}{\Delta v^R}\right] - \frac{\phi^*}{f(x_L)} \frac{v_\tau^L}{\Delta v^L} \left[\frac{v_{\tau y}^L}{v_\tau^L} - \frac{\Delta v_y^L}{\Delta v^L}\right]$$

Using the fact that $\psi\left(t^*, \phi^*, q\right) = 0$, we know that $\phi^* \frac{v_\tau^L}{\Delta v^L} = -\left(1 - \phi^*\right)\frac{v_\tau^R}{\Delta v^R}$, and so:

$$\psi_q = \frac{(1-\phi^*)}{f(y_R)} \frac{v_\tau^R}{\Delta v^R} \left[\left(\frac{v_{\tau y}^R}{v_\tau^R} + \frac{v_{\tau y}^L}{v_\tau^L} \cdot \frac{f(y_R)}{f(y_L)}\right) - \left(\frac{\Delta v_y^R}{\Delta v^R} + \frac{\Delta v_y^L}{\Delta v^L} \cdot \frac{f(y_R)}{f(y_L)}\right)\right]$$

Notice that the term outside the square brackets is negative (since $v_\tau^R < 0$), and the terms in the second parentheses within the square brackets are both negative (since $\Delta v_y^i = \int v_{\tau,y}^i < 0$ by the Spence-Mirrlees condition). Then, to show that $\psi_q < 0$, it suffices to show that the first parenthesis is positive. Now, since F is right-skewed*, $\frac{f(y_R)}{f(y_L)} < 1$. Moreover, we know that $\frac{v_{\tau y}^R}{v_\tau^R} > 0 > \frac{v_{\tau y}^L}{v_\tau^L}$, and so it suffices that $\frac{v_{\tau y}^R}{v_\tau^R} + \frac{v_{\tau y}^L}{v_\tau^L} > 0$, or equivalently that $v_\tau^L v_{\tau y}^R + v_\tau^R v_{\tau y}^L = \frac{\partial}{\partial y}[v_\tau^L v_\tau^R] < 0$. Now, again using the fact that $\phi^* \frac{v_\tau^L}{\Delta v^L} = -\left(1 - \phi^*\right)\frac{v_\tau^R}{\Delta v^R}$, we have: $v_\tau^L v_\tau^R = -\frac{1-\phi^*}{\phi^*}(v_\tau^R)^2 \frac{\Delta v^L}{\Delta v^R}$ and so:

$$\frac{\partial}{\partial y}[v_\tau^L v_\tau^R] = -\frac{1-\phi^*}{\phi^*}\left[\frac{\Delta v^L}{\Delta v^R} \cdot 2 v_\tau^R v_{\tau y}^R + (v_\tau^R)^2 \frac{\partial}{\partial y}\frac{\Delta v^L}{\Delta v^R}\right]$$

Clearly this will be negative provided that the square bracket is positive. The first term in the brackets is positive (since $\Delta v^i > 0$, $v_\tau^R < 0$ and $v_{\tau y}^R < 0$. The second term will also be positive provided $\frac{\partial}{\partial y}\left(\frac{\Delta v^L}{\Delta v^R}\right) > 0$ which implies $\frac{\Delta v_y^R}{\Delta v^R} > \frac{\Delta v_y^R}{\Delta v^R}$. This is implied by the

log-concavity of v in y. (To see this, note that $\frac{\Delta v_y^R}{\Delta v^R} - \frac{\Delta v_y^L}{\Delta v^L} = \int_{y_L}^{y_R} \frac{\partial^2 \log(\Delta v(t,y))}{\partial y^2} < 0$). Hence $\psi_q < 0$. $\qquad \square$

Proof of Proposition 6. First, we show that there exists a unique pair $(\hat{\kappa}, \hat{\rho})$ s.t. $\frac{F^{-1}(\hat{\rho})}{\overline{y}_F} = \hat{\kappa} = \frac{G^{-1}(\hat{\rho})}{\overline{y}_G}$. First we prove existence. Suppose $\frac{G^{-1}(p)}{F^{-1}(p)} > \frac{\overline{y}_G}{\overline{y}_F}$ for all $p \in (0,1)$. Then $\overline{y}_G = \int_0^1 G^{-1}(p)dp > \frac{\overline{y}_G}{\overline{y}_F} \cdot \int_0^1 F^{-1}(p)dp = \overline{y}_G$, which cannot be. Similarly, it cannot be that $\frac{G^{-1}(p)}{F^{-1}(p)} < \frac{\overline{y}_G}{\overline{y}_F}$ for all $p \in (0,1)$. Hence, there exists $\hat{\rho}$ s.t. $\frac{G^{-1}(\hat{\rho})}{F^{-1}(\hat{\rho})} = \frac{\overline{y}_G}{\overline{y}_F}$, or equivalently, $\frac{F^{-1}(\hat{\rho})}{\overline{y}_F} = \frac{G^{-1}(\hat{\rho})}{\overline{y}_G} = \hat{\kappa}$.

Next, we show uniqueness. Since G is more unequal than F, $\frac{G^{-1}(q)}{F^{-1}(q)} > \frac{G^{-1}(p)}{F^{-1}(p)}$, for every $0 < p < q < 1$. Hence, $\frac{G^{-1}(p)}{F^{-1}(p)} > \frac{\overline{y}_G}{\overline{y}_F}$ for all $p > \hat{\rho}$. Similarly, $\frac{G^{-1}(p)}{F^{-1}(p)} < \frac{\overline{y}_G}{\overline{y}_F}$ for all $p < \hat{\rho}$. Hence, $\hat{\rho}$ is unique.

We prove the first case $(F(y_R^F) \leq \hat{\rho})$ in the statement of the Proposition. The remaining cases are analogous. Let $\psi(\tau, \phi; y_L, y_R) = \phi \frac{v_\tau(\tau, y_L)}{\Delta v(\tau, y_L)} + (1-\phi) \frac{v_\tau(\tau, y_R)}{\Delta v(\tau, y_R)}$. By construction $\psi(t_F^*, \phi_F^*; y_L^F, y_R^F) = 0 = \psi(t_G^*, \phi_G^*; y_L^G, y_R^G)$. By the concavity of v, ψ is decreasing in τ. Similarly, by the log-submodularity of Δv, and since $y_L < y_R$, ψ is increasing in ϕ. Now, since $F(y_R^F) < \hat{\rho}$, $\frac{y_L^G}{y_G} < \frac{y_L^F}{y_F}$ and $\frac{y_R^G}{y_G} < \frac{y_R^F}{y_F}$. Thus, by the log-submodularity of Δv, $\frac{v_\tau(\tau, y_i^F)}{\Delta v(\tau, y_i^F)} \leq \frac{v_\tau(\tau, y_i^G)}{\Delta v(\tau, y_i^G)}$, for each $i \in \{L, R\}$. Hence $\psi(t_F^*, \phi_F^*; y_L^G, y_R^G) \geq 0$.

Suppose, $t_G^* < t_F^*$. Then $\psi(t_G^*, \phi_F^*; y_L^G, y_R^G) > 0$, since ψ is decreasing in τ. Hence, $\phi_G^* < \phi_F^*$, since $\psi(t_G^*, \phi_G^*; y_L^G, y_R^G) = 0$, and ψ is increasing in ϕ. But for each distribution $D \in \{F, G\}$, $\phi_D^* = \Pi[D(\overline{y}_D \cdot e'(t_D^*))] = P_D(\overline{y}_D \cdot e'(t_D^*))$. Now, since e is strictly concave, e' is strictly decreasing, and so $t_G^* < t_F^*$ implies $e'(t_F^*) < e'(t_G^*)$. By the single-crossing property, this in turn implies that: $F(\overline{y}_F \cdot e'(t_F^*)) < G(\overline{y}_G \cdot e'(t_G^*))$, which implies that $\phi_F^* < \phi_G^*$, since Π is strictly increasing. But this is a contradiction, and so $t_G^* \geq t_F^*$. $\qquad \square$

Proof of Lemma 1 and Corollary 1. Given Proposition 6, it suffices to show that $\hat{\kappa} > 1$, so that $F(y_R^F) < \hat{\rho}$. Since G is more right-skewed* than F, we know that $F(\overline{y}_F) < G(\overline{y}_G)$. Define $\hat{F}(k) = F(k \cdot \overline{y}_F)$ and $\hat{G}(k) = F(k \cdot \overline{y}_F)$. We have $\hat{F}(1) < \hat{G}(1)$. We showed in the proof of 6 that if G is more unequal than F, then $\hat{G}$ is a mean preserving simple spread of $\hat{F}$. Since $\hat{F}(1) < \hat{G}(1)$, the intersection of $\hat{F}$ and $\hat{G}$ must be at $\hat{\kappa} > 1$. $\qquad \square$

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B Endogenous Labor Supply (Online Appendix)

In this section, we consider a micro-founded version of the model, analogous to Meltzer and Richard (1981), in which agents supply labor elastically, and dead-weight losses arise endogenously as a consequence of labor market distortions. In doing so, we note that micro-foundations will affect the results only insofar as they change salient properties of the indirect utility function $v(\tau, x)$. Absent such changes, all of the results from sections 3 and 4 will continue to hold. Hence, it suffices to check the properties of $v(\tau, x)$.

There is a unit mass of agents. Each agent is characterized by their productivity x which is an i.i.d. draw from some distribution F. From herein, we refer to an agent's productivity as their type. Agents have preferences $u(c) + w(l)$ defined separably over consumption c and leisure l. Utility is increasing in both consumption and leisure ($u' > 0$ and $w' > 0$), and u and w both are concave, with at least one strictly concave. Agents are endowed with one unit of time, which they may allocate between leisure and work effort n. For simplicity, we assume $\lim_{l \to 0} w'(l) = \infty$, which rules out the corner solution in which some agent spends all of her time working. Agents supply their labor in competitive labor markets, and earn a wage equal to their productivity. Hence, the income of an agent with productivity x is $y = xn$.

The government levies a proportional tax τ on labor income that finances a lump-sum transfer T to each agent. Given the government policy (τ, T), the consumption of a type-x agent is: $c = T + (1 - \tau) xn$. The agent's problem is to choose the quantity of labor to supply to maximize:

$$\max_{n \in [0,1]} u\left(T + (1 - \tau) xn\right) + w\left(1 - n\right)$$

Given that preferences are strictly concave, the problem has a unique maximizer $\hat{n}(\tau, T; x)$. The maximizer is the solution to the first order condition:

$$(1 - \tau) xu'\left(T + (1 - \tau) xn\right) - w'\left(1 - n\right) \leq 0$$

with strict equality unless $\hat{n} = 0$. This will occur if $(1 - \tau) xu'(T) - w(1) < 0$, which implies that:

$$x < \frac{1}{1 - \tau} \cdot \frac{w'(1)}{u'(T)} = x_0(\tau, T)$$

Hence, all but the least productive agents will work. It is easily verified that work-effort is

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decreasing in the size of the transfer (i.e. $\frac{\partial \hat{n}}{\partial T} \le 0$, with strict inequality whenever $x > x_0$[29]), which implies that leisure is a normal good. Let $\hat{y}(\tau, T; x) = x\hat{n}(\tau, T; x)$ denote the income of a type-x agent. Notice that $\frac{\partial \hat{y}}{\partial x} = \frac{(1-\tau)xu'(\hat{c}) - nw''(\hat{l})}{-D} > 0$, and so agents' incomes are monotone in their productivity.

The average income in the economy is: $\overline{y}(\tau, T) = \int_0^\infty \hat{y}(\tau, T; x)dF(x)$. Since the government policy must be feasible, we have $T = \tau\overline{y}(\tau, T)$. Intuitively, the government budget constraint establishes a feasible level of transfers $T(\tau)$ for each level of taxes τ.[30] Hence, the government's redistribution policy amounts to the choice of a tax rate τ. Moreover, we assume that households understand that government policy is subject to its budget constraint; there is no fiscal illusion. Accordingly, let $n(\tau; x) = \hat{n}(\tau, T(\tau); x)$ and $y(\tau; x) = \hat{y}(\tau, T(\tau); x)$ be the labor supply and income of a type-x agent, given tax rate τ and the associated transfer $T(\tau)$. Similarly, let $\overline{y}(\tau) = \int_0^\infty y(\tau; x)dF(x)$ denote the average income, given tax rate τ and the associated transfer $T(\tau)$.

Let $v(\tau, x)$ denote the indirect utility function of a type x agent. We have:

$$v(\tau, x) = u\left(\tau\overline{y}(\tau) + (1 - \tau)xn(\tau, x)\right) + w\left(1 - n(\tau, x)\right)$$

We seek to establish the parallels between the properties of the indirect utility functions from the structural and reduced-form approaches. By the envelope theorem, $v_\tau(\tau, x) = \left[\frac{\partial \tau\overline{y}}{\partial \tau} - y(\tau; x)\right]u'(c(\tau, x))$. This is directly analogous to the correponding expression (equation 1) in the reduced-form model.[31]

The indirect utility function does not generically inherit the curvature properties of the direct utility function. In particular, v need not be concave in τ. However, we have the following result:

Lemma 2. *The indirect utility function $v(\tau, x)$ is pseudo-concave in τ for each x.*

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[29]To see this, let $D = (1 - \tau)^2 x^2 u''(\hat{c}) + w''(\hat{l})$. By the strict concavity of preferences, $D < 0$. Applying the implicit function theorem to the first order condition gives: $\frac{\partial n}{\partial T} = \frac{(1-\tau)xu''(\hat{c})}{-D} < 0$.

[30]To see this formally, fix any $\tau \in [0, 1]$. Define the function $\psi(T; \tau) = \tau\overline{y}(\tau, T) - T$. Notice that $\psi(0; \tau) > 0$ and $\lim_{T\to\infty} \psi(T; \tau) < 0$ and $\frac{\partial \psi}{\partial T} = \tau\frac{\partial \overline{y}}{\partial \tau} - 1 < 0$, since $\frac{d\overline{y}}{dT} < 0$. The result follows by the intermediate value theorem.

[31]In fact, if the reduced-form function $e(\tau)$ satisfies $e'(\tau) = 1 + \frac{\partial \overline{y}}{\partial \tau} \cdot \frac{\tau}{\overline{y}} = 1 + \varepsilon(\tau)$, where $\varepsilon(\tau)$ is the average tax elasticity of labor supply, then the expressions for the marginal utility of taxation are identical across the two models. For example, if preferences are given by $c - \frac{\theta}{1+\theta}(1 - l)^{\frac{1+\theta}{\theta}}$ (as in Greenwood, Hercowitz and Huffman (1988) and Correia, Neves and Rebelo (1995), amongst others) then the tax elasticity of labor supply is a constant $\varepsilon(\tau) = -\theta$, and so the implied reduced-form dead-weight loss function is $e(\tau) = (1 + \tau)\theta + \theta\ln(1 - \tau)$.

The pseudo-concavity of $v(\tau, x)$ guarantees that the first order conditions characterize the optimal tax rate for each voter. Analogous to the reduced-form case, any voter whose income would be above average when there is zero taxation will prefer zero taxation, and all other agents will demand a positive level of redistribution, with the optimal tax rate satisfying: $\overline{y} + \tau \frac{\partial \overline{y}}{\partial \tau} - y(\tau; x) = 0$. This condition is identical to equation (13) in Meltzer and Richard (1981), which defines the optimal tax rate in their framework.[32]

Finally, we establish that a condition, analogous to Assumption 1 ensures that agents' preferences over tax policies satisfy the Spence-Mirrlees condition. First note that:

$$v_{\tau x} = \left[(1 - \tau) \left(\frac{\partial \tau \overline{y}}{\partial \tau} - y(\tau, x) \right) u''(c(\tau, x)) - u'(c(\tau, x)) \right] \frac{\partial y(\tau, x)}{\partial x}$$

Since $\frac{\partial y(\tau, x)}{\partial x} > 0$, the sign of $v_{\tau x}$ depends on the sign of the term in square brackets. Now, as in the reduced-form case, the condition is guaranteed to be satisfied for agents with incomes that are sufficiently low (i.e. if $y(\tau, x) < \frac{\partial \tau \overline{y}}{\partial \tau}$). For agents with larger incomes, the Spence-Mirrlees condition is satisfied provided that:

$$R(c^*) < \frac{(1 - \tau) y(\tau, x) + \tau \overline{y}(\tau)}{(1 - \tau) y(\tau, x) - (1 - \tau) \frac{\partial (\tau y)}{\partial \tau}}$$

which is analogous to Assumption 1. The marginal utility of taxation is monotone in agents' incomes provided that the coefficient of relative risk aversion is not too large for high productivity agents. It is easily shown that this assumption is equivalent to the assuming that $\frac{\partial n}{\partial \tau} < 0$ for all τ and all x.[33]

As we briefly noted in section 2, when labor supply is elastic, imposing the Spence-Mirrlees condition is equivalent to assuming that taxation deters work effort. To make sense of this, note that increasing labor taxes (whilst simultaneously increasing transfers) has two effects. The substitution effect unambiguously deters work effort, whilst the sign of the wealth effect is ambiguous. Since a tax increase is combined with an increase in transfers, the wealth effect (further) deters work effort whenever $y(\tau, x) < \frac{\partial \tau \overline{y}}{\partial \tau}$, and stimulates it otherwise.[34] The Spence Mirrlees condition implies that, for high productivity agents, the wealth effect cannot

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[32]Pseudo-concavity does not guarantee that the problem admits a unique maximizer, although the set of optimizers is guaranteed to be convex. We follow Meltzer and Richard (1981) in assuming a unique solution.

[33]To see this, note that $\frac{\partial n}{\partial \tau} = x \frac{\left[(1-\tau)\left(\overline{y} + \tau \frac{\partial \overline{y}}{\partial \tau} - y(\tau, x)\right) u''(c^*) - u'(c^*) \right]}{-D}$. Hence $v_{\tau x} = \frac{(-D)}{x} \cdot \frac{\partial y}{\partial x} \cdot \frac{\partial n}{\partial \tau}$. Then since $D < 0$ and $\frac{\partial y}{\partial x} > 0$, then $sign(v_{\tau x}) = sign\left(\frac{\partial n}{\partial \tau}\right)$.

[34]To see this, first note that absent the increase in transfers, the Slutsky Equation implies: $\frac{\partial n}{\partial \tau} = \frac{\partial n^h}{\partial \tau} - y \frac{\partial n}{\partial T}$. Adding the increased transfers gives an overall effect: $\frac{\partial n^h}{\partial \tau} - \left(y - \frac{\partial \tau \overline{y}}{\partial \tau}\right) \frac{\partial n^*}{\partial T}$.

be so large as to overwhelm the substitution effect. Following an increase in taxes, all agents work less.

Thus, we have shown, with the exception of concavity, all salient features of the indirect utility function are implied by micro-foundations. The fact that $v(\tau, x)$ is not guaranteed to be strictly concave is unfortunate, but not fatal. We have already shown that the optimal tax rate is characterized by the first order conditions, notwithstanding the failure of concavity. The other main role played by concavity was in guaranteeing that the bargaining game admitted an equilibrium in no delay.[35] To achieve this result, concavity is sufficient, but not necessary. No-delay equilibria require that, for every agent, there is *some* decisive coalition C including that agent, for which the associated coalitional acceptance set A_C is non-empty. Concavity of preferences guaranteed that the acceptance set for *every* coalition is non-empty – which is clearly stronger that necessary. It suffices, for example, that the acceptance set associated with the smallest connected coalition containing a given agent be non-empty. With continuous preferences, this may plausibly be the case, even when some agents' preferences are non-concave, provided that they are not too convex.

Proof of Lemma 2. To prove the pseudo-concavity of the indirect utility function it suffices to show that, for $\tau \in (0, 1)$, if $v_\tau(\tau^*, x) = 0$ then v_τ achieves a maximum at τ^*. Suppose $v_\tau(\tau^*, x) = 0$ for some $x > 0$. Recall $v_\tau(\tau, x) = \left[\frac{\partial \tau \bar{y}}{\partial \tau} - y(\tau; x)\right] u'\left(c(\tau, x)\right)$ and that income y is monotonically increasing in productivity x. Thus, whenever $x' < x$, then $v_\tau(\tau^*, x') > 0$. Similarly, whenever $x'' > x$, $v_\tau(\tau^*, x'') < 0$.

Takes some small $\varepsilon > 0$. Since v is continuously differentiable, it follows that $v_\tau(\tau^* - \varepsilon, x') > 0$ for ever $x' < x$. By continuity, this implies that $v_\tau(\tau^* - \varepsilon, x) \geq 0$. Similarly, $v_\tau(\tau^* + \varepsilon, x) \leq 0$. Together, these rule out τ^* as a minimizer (which would require $v_\tau(\tau^* - \varepsilon, x) < 0$) or as a saddle point (which would require that $v_\tau(\tau^* - \varepsilon, x)$ and $v_\tau(\tau^* + \varepsilon, x)$ have the same sign. $\square$

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[35]We also used concavity to establish that the social acceptance set was an interval bounded by the certainty equivalents of the left and right decisive voters. Absent concavity, the social acceptance set is not guaranteed to be an interval. However, the boundaries of the convex hull of the acceptance set will continue to be the respective certainty equivalents, and these will continue to converge to the same limit as in Proposition 3.