Bargaining and Strategic Voting on Appellate Courts

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We *explore the properties of voting rules and procedures employed by appellate courts in the US. Our model features: (1) a two-stage decision-making process (first over case disposition, then over majority opinion content), (2) dispositional consistency, (3) restricted bargaining entrée, (4) competitive majority opinions, and (5) absolute majority in joins. We show that the median judge is pivotal over case dispositions, although she (and others) may not vote sincerely. Strategic voting becomes more likely as the location of the case becomes more extreme, resulting in majority coalitions that give the appearance of less polarization on the court, than is truly the case. The equilibrium policy generically does not coincide with the ideal policy of the median judge either in the dispositional majority or the bench as a whole. Rather, opinions are drawn toward a weighted center of the dispositional majority but often reflect the preferences of the opinion author.*

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INTRODUCTION

Many decision-making bodies process cases. Some, such as admissions committees, permitting agencies, and trial courts produce a single output for each case, a case disposition or judgment. Thus, the committee admits or rejects the applicant, the agency grants or denies the permit, and the adjudicator rules for plaintiff or defendant. Other bodies, however, produce *two* outputs for each case, not one. The first output remains the case disposition, but the second is a general rule or policy that rationalizes the judgment in the case. Examples of institutions that jointly produce judgments and policies include the U.S. Supreme Court, the U.S. Courts of Appeal, supreme courts in the American states, many American regulatory commissions and bodies, high constitutional courts around the world as they engage in ex-post review of statutes, and some international courts such as the European Court of Justice, among others.

In this paper, we present a new model of the joint production of judgments and policies by multi-member bodies like the U.S. Supreme Court. Our model of joint production, which we call the 'American practice', captures a stylized version of procedures ubiquitous in American appellate courts and regulatory commissions. In our formalization, the court first produces a judgment, then produces a rule that rationalizes the judgment. Members of the winning dispositional coalition bargain over the rule using procedures that we model as a sequential bargaining game. Although the model reflects the American practice, the techniques employed can be adapted to study decision making on other joint-production institutions that use somewhat different practices.

We find that joint production using the American practice strongly affects policies and may affect case dispositions. First, we show that in equilibrium the median judge is pivotal over case dispositions. However, judges face an incentive to vote strategically on case dispositions in order to join the bargainers who determine policy. Equilibrium dispositional coalitions are nonetheless connected, as the most extreme judges are the least likely to vote strategically. By contrast, moderate judges, in order to affect policy, face a strong incentive to vote contrary to their preferred outcome. In particular, although the median judge is dispositionally pivotal, she may nevertheless vote strategically: the majority disposition need not always coincide with the median judge's more preferred disposition. Case locations also affect the incentive to vote strategically on dispositions, as it is more attractive to do so when the case is close to the judge's ideal rule cut-point rather than far from it.

Second, we show that the assigned author has considerable latitude in setting policy when bargaining is not intense (i.e. when the discount parameter δ in the sequential bargaining subgame is far from 1). In these circumstances, the opinion author, though constrained by the case location, will be able to set policy at or near her ideal policy. As bargaining becomes intense (i.e. as δ approaches 1), the dispositional majority endogenously separates into two factions. In these circumstances, the announced rule is either the ideal policy of some pivotal judge (not necessarily the median of the dispositional majority), or the result of asymmetric Nash Bargaining between representative leaders of the factions, with bargaining weights proportional to faction size. Importantly, in the limit, the chosen policy never coincides with the ideal policy of the median judge of the whole court; thus while the median judge decides the disposition of the case, she does not determine the policy of the court. This result contrasts starkly with both the median voter results and median-of-the-majority results that have been proposed in the existing literature. In fact, many of the model's predictions are new to the literature on appellate courts and regulatory commissions. Many can be taken to data.

Reaching these results requires a contribution to the theory of sequential bargaining games. In particular, the voting rule for policy implies varying majority quotas depending on the size of the majority dispositional coalition. Consequently, we characterize the equilibria of uni-dimensional sequential spatial bargaining games for any super-majority quota. An analytic contribution of this paper is characterizing the limit equilibria of such bargaining games as the discount parameter δ approaches 1, which we interpret as the limit as the cost of proposing alternative majority opinions becomes arbitrarily small —i.e. as bargaining becomes intense. We believe this situation is particularly relevant given the institutional setting of an apex court.

Literature Review

Formal models of high courts have struggled to accommodate the joint production of judgments and policy. Early models discarded judgments by treating appellate courts as "little legislatures" (Hammond et al. (2005), Jacobi (2009)). These models often invoked the median voter theorem despite the absence of Condorcet-compatible procedures on high appellate courts. Others, responding to the wide-spread observation that majority opinions often reflect the policy views of the opinion author, invoked the Romer and Rosenthal (1978) monopoly agenda setter model (Hammond et al. (2005), Lax and Cameron (2007)). The source and limits of the opinion author's monopoly power remained murky since any justice in the dispositional majority can offer a competing opinion and sometimes does.

A second group of models discarded policy to focus on judgments, effectively treating appellate courts as "big juries" (big in the sense of singularly important) (Fischman (2011), Iaryczower and Shum (2012), Lax (2007)). Despite the extremely interesting insights that follow, this approach jettisons much of what makes high appellate courts notable.

Recent papers more forthrightly address the joint production of judgments and rules. Carrubba et al. (2012) stands out as particularly innovative and insightful. This paper treated judgment and policymaking as a two-stage sequence, and noted the importance of restricted bargaining entrée. In the bargaining subgame, it focused on the core in a pure majority rule voting game within the dispositional majority, sidestepping policy consistency and the absolute-majority-in-joins rule. The model neatly rationalized the empirical finding that majority opinion location is often far from the location of the median justice of the whole Court and depends strongly on the case disposition (Clark and Lauderdale (2010)). The model, however, left little room for author influence on opinion location. Nor did it address strategic dispositional voting despite the seemingly strong incentive for judges to join the dispositional majority in order to affect the bargaining outcome. Finally, preferences over judgments and policies were somewhat *ad hoc*.

The current paper, following Carrubba et al. (2012), adapts the judgment-policy sequence and restricted bargaining entrée (explained below). In addition, it explicitly addresses rule consistency, the absolute-majority-in-joins voting rule, the implications of the designated proposer procedure, and the possibility of strategic dispositional voting. Judicial preferences over judgments and policy are tightly linked and internally consistent. These extensions allow the distinctive logic and empirical implications of the American procedure to emerge more clearly than heretofore.

THE MODEL

The American Practice

A stylized version of the American practice for jointly producing judgments and policies has the following features:

1. Production is sequential. The first stage is the judgment, the second policy selection.
2. The actors —whom we call judges —determine the judgment using pure majority rule. A minority-side dispositional vote is referred to as a "dissent."
3. Judges in the dispositional majority, and only those judges, bargain over policy selection. We call this property "restricted bargaining entrée" (RBE).
4. The policy selected by the dispositional majority must yield the judgment determined by the dispositional vote, when applied to the case. We call this property "rule consistency".
5. A designated opinion author offers a policy for consideration by the dispositional majority. The judges do not modify the designated author's proposal using a legislative-style amendment tree. Rather, each judge in the dispositional majority endorses the designated proposer's policy ("join") or declines to endorse the policy ("concur").
6. To have precedential value, a policy must secure an absolute majority in endorsements from the full Court, not just a majority within the dispositional majority. We call this unusual voting rule

"absolute-majority-in-join" (AMJ). AMJ implies that the effective voting quota in the dispositional majority depends on the size of the dispositional majority, and may range from pure majority rule to unanimity.

7. Failure by the designated opinion author to secure an absolute majority in joins may trigger another judge to propose a policy that can secure an absolute majority in joins.

Actual procedures vary somewhat from institution to institution and even from case to case.[1] Nonetheless, these seven features arguably capture the essence of the American practice. As a robustness check, we show below that weakening features (1), (3), and (5) does not significantly change the results of the model.

Cases, Dispositions and Rules

A court consisting of n judges (where n is odd) must decide a case. A *case* z encodes details of an event that has occurred, for example, the level of care exercised by a manufacturer or the intrusiveness of a police search. Let $Z = [0, 1]$ be the case space. A judicial *disposition* $d \in \{0, 1\}$ determines which party to the dispute prevails.

Judges dispose of cases by applying a *legal rule*. A legal rule $r : Z \to \{0, 1\}$ assigns a disposition to every potential case. We focus on an important class of legal rules, *cutpoint*-based doctrines, which take the form:

$$r(z; y) = \begin{cases} 1 & \text{if } z > y \\ 0 & \text{if } z < y \end{cases}$$

where y denotes the cutpoint. For example, in the context of negligence, the defendant is not liable

[1]For example, on the U.S. Supreme Court the initial vote on the disposition is a straw vote used for determining opinion assignment. On rare occasions, initial dissenters switch and enter the majority dispositional coalition.

if she exercised at least as much care as the cutpoint y.[2] Although cutpoint rules can be summarized by a threshold case, it should be clear that rules and cases are fundamentally different objects.

Decision Making by the Court

Decision-making by the justices occurs in two stages. In the first stage, each judge casts a dispositional vote ($d^j \in \{0, 1\}$), and the disposition of the court is determined by simple majority rule. The dispositional votes of each judge separate the judges into dispositional majority (denoted $M \subset \{1, .., n\}$) and minority coalitions. Necessarily, $|M| \geq \frac{n+1}{2}$.

In the second stage, the justices in the dispositional majority must agree upon a legal rule y that rationalizes the chosen disposition. Consistency requires that $y \leq z$ if $d = 1$ and $y \geq z$ otherwise.[3] In the baseline model, we assume that the once the first-stage dispositional majority is determined, it remains fixed. In Appendix A.2, we present an extension in which the judges may change their dispositional votes after observing the majority (and dissenting) opinions. With one modification, our baseline results continue to hold in this alternative setting.

The judges in the dispositional majority bargain over which legal rule to implement. We formalize this by studying a bargaining framework á la Baron and Ferejohn (1989) and Banks and Duggan (2000). Initially, a judge from the dispositional majority is recognized to propose a policy y, consistent with the majority's disposition. Upon seeing the proposal, each judge in the dispositional majority either endorses the proposed opinion by 'joining' or declines to endorse the opinion by 'concurring'. To become the policy of the court, the proposal requires the assent of a majority *of the*

[2]Other examples include allowable state restrictions on the provision of abortion services; state due process requirements for death sentences in capital crimes; the degree of procedural irregularities allowable during elections; and the required degree of compactness in state electoral districts.

[3]For technical reasons, we require the weak inequality in both cases. We could make one inequality strict by discretizing the policy space.

entire court, not just the dispositional majority. Thus, in many cases, the dispositional majority will bargain under an effective super-majority rule.[4] If the proposal is accepted, it is implemented and the bargaining game ends. Else, the process repeats itself in the following period, and this continues until a policy of the court emerges. Delay within the bargaining game is costly, and the judges share a common discount factor $\delta \in [0, 1)$.

In the first period of bargaining, we allow the identity of the proposing judge to be non-random, reflecting the current practice where the most senior judge in the dispositional majority determines who will write the opinion. However, in subsequent bargaining periods, we assume judges are randomly recognized with uniform probability, reflecting the equal right of every justice to counter-propose policies.[5]

Judicial Preferences

Following Carrubba et al. (2012) and Cameron and Kornhauser (2008), we assume that judges' preferences exhibit both *expressive* and *policy* components.[6] Policy utility depends on the actual policy implemented by the dispositional majority, reflecting the judge's concern for how future

[4]Intuitively, no judge in the dispositional minority will endorse the proposal; doing so would require them to support a policy inconsistent with their dispositional vote.

[5]None of the results in the policy-making stage (section 3) turn on the uniformity assumption. As the proof of Proposition 1 attests, our analysis of policy-making can easily accommodate arbitrary recognition probabilities. In the adjudication stage (section 4), our results will hold provided that, adding a judge to a coalition doesn't skew the relative recognition probabilities of existing coalition members by too much. Formally, for two coalitions C, C' and two judges $i, j \in C \cap C'$, $\frac{p_i^C}{p_j^C}$ and $\frac{p_i^{C'}}{p_j^{C'}}$, are not too different.

[6]The motivations of judges has sparked large normative and positive literatures, as well as statements by judges themselves (see, inter alia, O'Brien (2016) and Posner (2010)). One may find support for almost any portrayal of judicial preferences. An exemplary recent empirical study is Ash and MacLeod (2015), finding evidence for intrinsic motivation in state Supreme Court judges.

cases will be decided. Expressive utility depends neither on the policy chosen, nor on the actual disposition of the case, but rather, on the judge's individual dispositional vote. Whereas policy preferences are consequentialist —they depend on actual outcomes —expressive preferences reflect the judge's desire to be seen to decide cases 'correctly', notwithstanding, how, if at all, their vote changes actual outcomes. As will become clear, absent an expressive component of utility, judges would never have an incentive to dissent. Rather than taking an *ad hoc* approach to specifying these preferences, we present a framework that makes sense of both components in a cohesive way. We first specify the dispositional preferences of a given judge, and build both expressive and policy preferences from this.

Suppose judge j has ideal threshold x^j, and that $0 \leq x^1 \leq .... \leq x^n \leq 1$, so that the judges are ordered by their ideal threshold. Judge j's *dispositional utility* is:

$$u_D(d; z, x^j) = \begin{cases} 0 & \text{if } d = r(z; x^j) \\ l(z - x^j) & \text{if } d \neq r(z; x^j) \end{cases}$$

where $l(\cdot)$ is a quasi-concave 'loss' function that satisfies $l(0) = 0$ and $l(\cdot) < 0$ otherwise (i.e. l has a single peak at 0). There is a cost to judges when the disposition is different to their ideal. The (strict) quasi-concavity of l implies that dispositional preferences satisfy the *increasing differences in dispositional values (IDID)* property (see Cameron et al. (2019)), which entails that the cost of making 'incorrect decisions' becomes larger the further is the case from the threshold x^j. Intuitively, judges feel more strongly about 'incorrectly' deciding 'clear-cut' cases (those far from the threshold), than 'contestable' ones (those close to the boundary separating acceptable and unacceptable conduct).[7]

The expressive component of a judge's utility is simply the dispositional utility associated with the outcome for which she votes. To construct policy utility, we assess the utility implications of

[7]Such preferences are common in the related literature. See Baker and Mezzetti (2012), Chen and Eraslan (2018), amongst others.

applying rule y to decide future cases. The judge's *policy* utility is her expected dispositional utility from having the next case decided according to rule y, given the distribution over cases that are likely to arise. Recall, $r(z, y)$ is the disposition that results from applying rule y to case z. We have:

$$u_P(y; x^j) = \int u_D(r(z, y); z, x^j)dF(z)$$

where cases are drawn from a continuous distribution $F(z)$ with density $f(z)$.

The *IDID* property implies that policy utility $u_P(y; x)$ is strictly quasi-concave in y for every x, although it is not necessarily concave. Moreover, the *IDID* property implies that, whenever $x^i > x^j$, $\frac{\partial u_P(y;x^i)}{\partial y} > \frac{\partial u_P(y;x^j)}{\partial y}$, or equivalently, $\frac{\partial^2 u_P(y,x)}{\partial x \partial y} > 0$.[8] Hence, preferences exhibit the single-crossing property; the benefit from marginally increasing the policy y is monotone in the judges' ideal policies.

Example 1. *Suppose cases are uniformly distributed on* $[0, 1]$. *Table 1 provides a mapping between the dispositional loss function l and commonly used policy utility functions, including absolute-value utility, quadratic utility, and bell-curve shaped (Gaussian density) utility. Bell-curve shaped policy utility has some nice properties that we exploit in later examples.* □

TABLE 1. Relationship between Dispositional and Policy Utility.

Dispositional Utility	Policy Utility
$l(z - x_i) = -1$	$u_i^P(y) = -\lvert y - x_i\rvert$
$l(z - x_i) = -\lvert z - x_i\rvert$	$u_i^P(y) = -\frac{1}{2}(y - x_i)^2$
$l(z - x_i) = -\lvert z - x_i\rvert e^{-\frac{1}{2}(z-x_i)^2}$	$u_i^P(y) = e^{-\frac{1}{2}(y-x_i)^2} - 1$

During the bargaining game, the disagreement payoff to each judge is $u_P(D; x)$. We make the standard assumption that disagreement is worse for each judge than agreeing to any feasible policy (i.e. $u_P(D, x) \leq u_P(y, x)\ \forall\ y \in [0, 1]$).

[8] To see this, note that, for any policy y and ideal policy x, $\frac{\partial u_P(y;x)}{\partial y} = (-1)^{\mathbf{1}[y<x]} l(y - x)$. Then, $\frac{\partial^2 u_P(y,x)}{\partial x \partial y} = (-1)^{\mathbf{1}[y>x]} l'(y - x) > 0$, since $l'(z) > 0$ if $z < 0$ and $l'(z) < 0$ otherwise.

Overall utility is the sum of policy and expressive components:

$$u_P(y; x^j) + \alpha u_D(d^j; z, x^j)$$

where $\alpha > 0$ denotes the relative importance of the expressive component. Notice that policy utility depends on the actual chosen policy y, whereas expressive utility depends only upon the judge's dispositional vote.

Our formulation of judicial preferences can be further motivated as follows: Consider a dynamic model in which the court confronts a single case in each future period, and suppose judges discount the future at rate $\rho \in (0, 1)$.[9] Take a given case z, and a rule y that decides the current and all future cases, each assumed to be an independent draw from distribution $F(z)$. The expected lifetime utility of a judge having purely consequentialist preferences is:

$$u_D(r(z; y); z, x^j) + \frac{\rho}{1-\rho} u_P(y; x^j)$$

Setting $\alpha = \frac{1-\rho}{\rho}$, this expression almost exactly coincides with our formulation of judicial utility. (Our formulation differs only in that current period utility depends on the judge's dispositional vote; not the actual disposition of the case.) Moreover, now, α has a natural interpretation as the importance to utility of the current case relative to the future stream. As $\alpha \to 0$, the court becomes perfectly future (and thus, policy) oriented, whereas as $\alpha \to \infty$, the court ignores the future entirely, and only cares about the current case.

We should note the role of the 'legal *status quo*' within the bargaining game. We take the view that any prior legal policy effectively reverts to a null policy when the Court takes the case – policy is in limbo until the Court resolves the case. Indeed, our bargaining protocol requires that bargaining continue until a majority policy is agreed to. Policy may only revert to the *status quo ante* if the

[9]For clarity, the discount factor δ captures the cost of delay in the bargaining phase when deciding a single case. The discount factor ρ reflects the judges' present bias, and the passage of time *between* cases.

Court re-enacts it anew. In Appendix A.1, we consider an alternative framework in which policy reverts to the *status quo ante* if bargaining fails. Our results continue to hold under this alternative formulation, and so the question of the 'legal *status quo*' is not crucial to our analysis.

Additionally, we note that our formulation implicitly assumes that the court can commit to implementing its chosen policy when deciding future cases; i.e. the announced policy is time-consistent and renegotiation-proof. Cameron et al. (2019) show that the *IDID* property is sufficient to sustain policy commitment in equilibrium, provided that judges are sufficiently future oriented. Rasmusen (1994) provides a similar analysis, although a different mechanism enforces commitment in his model. Beyond these, we know of no other models of collegial courts that address the problem of commitment. Recent legislative models of sequential policy making with evolving *status quos* are suggestive (Baron (1996), Kalandrakis (2010)) but we do not pursue this point any farther.

Strategies and Equilibrium

We analyze equilibrium in the policy-making stage and the dispositional-voting (adjudication) stage, in turn.

Given the repeated game structure of bargaining in the policy-making stage, strategies can be quite complex as they may be history dependent. We restrict attention to stationary strategies, which require that players choose equivalent strategies in every structurally equivalent sub-game. A strategy for judge j (in the dispositional majority) in the policy-making stage is a pair $\left(y^j, A^j\right)$, where:

- $y^j(z, M, \delta)$ denotes the policy proposed whenever the judge is recognized to make a proposal, given case z and dispositional majority $M \subset \{1, .., n\}$.
- $A^j(z, M, \delta)$ denotes the set of proposals that the judge will accept, whenever she is in the dispositional majority.

The equilibrium concept is stationary sub-game perfection with weakly undominated strategies. Weak undominance requires that each judge vote for her more preferred option (regardless of whether her vote would sway the outcome or not). This rules out equilibria in which judges vote for less favored outcomes, sustained by the belief that their vote will be inconsequential.

A strategy for judge j at the adjudication stage is a dispositional choice $d^j(z; \alpha, \delta) \in \{0, 1\}$ given case z, anticipating the equilibrium rule that will be chosen in the policy-making stage. An adjudication (Nash) equilibrium is a pair (d, M) denoting the majority disposition and the composition of the dispositional majority, having the property that no judge could do better by switching her dispositional vote.

THE POLICY STAGE

In this section we characterize behavior in the policy-making stage for a generic dispositional majority. In section 4, we find the optimal majority coalition, given the policy bargaining that is anticipated to follow.

We begin by characterizing equilibrium proposals when $\delta < 1$. As we will see, there will be a range of policies proposed in equilibrium, reflecting the agenda-setting prerogative of the opinion author. Our approach is, thus, distinct from median-voter-type models that predict a single equilibrium policy. We subsequently reconcile the two approaches by taking the limit as the agenda-setter's power goes to zero. Even in this scenario, the equilibrium policy will not generically coincide with the median judge's ideal.

Equilibrium Characterization

Let z be the case, and suppose the dispositional majority coalition $M \subseteq \{1, ..., n\}$ contains $m \in \{k, ..., n\}$ members, where $k = \frac{n+1}{2}$. Without confusion, we re-label the judges in the coalition, preserving the ordering of ideal policies, so that $M = \{1, ..., m\}$ with $x^1 \leq .... \leq x^m$. Given the two-stage structure, once the majority coalition has been determined, the preferences of the non-majority judges become inconsequential to policy-making, so we are free to disregard them, and focus on the m remaining judges. Similarly, we may focus solely on policy utility, since dispositional utility was determined at the time of the dispositional vote.

Recall that policy must be consistent with the disposition of the court. If the majority disposition was 1, the majority must choose a policy in the interval $[0, z]$, whilst if the disposition was 0, it must choose a policy in the interval $[z, 1]$. Generically, the court's policy must be contained in $\left[\underline{x}, \overline{x}\right]$, where $\underline{x} \in \{0, z\}$ and $\overline{x} \in \{z, 1\}$.

Our bargaining framework is similar to those studied by Banks and Duggan (2000), Cardona and Ponsati (2011) and Parameswaran and Murray (2019). Since those papers provide detailed expositions of the equilibrium characterization, we defer to them, and instead provide a brief intuitive account of the equilibrium.

We begin by noting two important details. First, each judge bases her decision to support a proposal or not by comparing the policy utility from the current proposal to her (discounted) expected policy utility from entertaining counter-proposals. The set of equilibrium counter-proposals, thus, establishes the opportunity cost of accepting a given proposal, which in turn establishes the set of proposals acceptable to each judge. Since each proposer seeks to build a winning coalition around their proposal, the anticipation of future counter-proposals disciplines each judge's decision about which policy to propose when they are recognized.

Second, because policy preferences satisfy the single-crossing property, in equilibrium, the policy coalitions that support and reject any proposal will both be connected. We stress that this is an equilibrium phenomenon; the decision rule does not require that the 'join' and 'concur' coalitions be connected, but optimal behavior, nevertheless, ensures that they will. Since the proposer only needs the support of $k = \frac{n+1}{2}$ judges, it suffices to either earn the support of the left-most k judges $\{1, ..., k\}$ in the dispositional majority, or the right-most k judges $\{m - k + 1, ..., m\}$, where judge $m - k + 1$ is the k^{th} judge from the right. It follows that judges $\{m - k + 1, ..., k\}$ must be in every equilibrium coalition. Indeed, judges $m - k + 1$ and k are *decisive* in the sense that, in equilibrium, a proposal is winning if and only if it has both their support. Following Compte and Jehiel (2010), we refer to these as the *left-* and *right-decisive judges*, respectively. If $m = n$, so that the join coalition need only be a simple majority of the dispositional majority, then the left- and right-decisive judges will both coincide with the median judge. By contrast, for any $m < n$, $m - k + 1 < k$, and so, generically, the decisive judges will be non-median players, with distinct preferences.

For notational convenience, we index the left and right decisive judges by l and r, where $l = m - k + 1$ and $r = k$. We have the following result, which is similar (although there are some important differences) to results previously noted by Cho and Duggan (2003), Cardona and Ponsati (2011), Parameswaran and Murray (2019), amongst others:

Proposition 1. *For $\delta < 1$, the bargaining game admits a unique equilibrium. The equilibrium is in no-delay, and is characterized by a pair $\left(\underline{y}, \overline{y}\right)$, with $\underline{x} \leq \underline{y} < \overline{y} \leq \overline{x}$, such that:*

1. *When recognized, judge j will propose:* $y^j = \begin{cases} \underline{y} & x^j < \underline{y} \\ x^j & x^j \in \left[\underline{y}, \overline{y}\right] \\ \overline{y} & x^j > \overline{y} \end{cases}$
2. *The pair $\left(\underline{y}, \overline{y}\right)$ satisfies:*

- $\underline{y} = \min\{y \geq \underline{x} | u_P(y; x^r) \geq (1 - \delta)u_P(D, x^r) + \frac{\delta}{m} \sum_j u_P(y^j, x^r)\}$
- $\overline{y} = \max\{y \leq \overline{x} | u_P(y; x^l) \geq (1 - \delta)u_P(D, x^l) + \frac{\delta}{m} \sum_j u_P(y^j, x^l)\}$

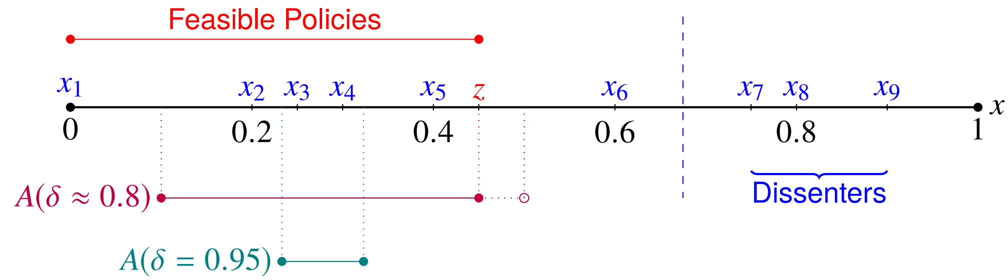
Proposition 1 shows that our bargaining game admits a unique equilibrium characterized by an interval $[\underline{y}, \overline{y}]$ of 'socially acceptable' policies (i.e. which will receive the support of at least k agents). In equilibrium, each judge will propose the socially acceptable policy closest to her ideal. Judges with 'moderate' preferences (whose ideal policies lie within the interval) will successfully implement their ideal rule in equilibrium, whereas judges with 'extreme' preferences must offer a compromise rule. All ‘extreme left’ judges will pool on the same proposal $\underline{y}$, whilst all ‘extreme right’ judges will pool on the same proposal $\overline{y}$. What constitutes ‘moderate’ and ‘extreme’ is itself determined in equilibrium, and depends on the discount factor δ, and the preferences of the left- and right-decisive judges. $\overline{y}$ is the highest policy that the left-decisive judge is willing to accept, given her continuation payoff. Similarly, $\underline{y}$ is the lowest policy that the right-decisive judge is willing to accept. Any proposal in the region $\left[\underline{y}, \overline{y}\right]$ is equilibrium consistent.

To make sense of Proposition 1, let $E[y] = \sum_j p_j y^j$ be the expected policy that will be proposed (and accepted) in the continuation game. As we show in Appendix B, if $E[y]$ is proposed, it will receive unanimous support.[10] Take a judge j in the dispositional majority whose ideal policy lies below $E[y]$. Since delay is costly ($\delta < 1$), judge j can offer a policy slightly below $E[y]$ and still retain unanimous support. Decreasing the offer further, she will eventually lose the support of the right-most judge, then the second-most-right judge, and so on. Since it suffices to have the support of the right-decisive judge, judge j will continue decreasing the offer until either she reaches her ideal policy, or the support of the right decisive judge would be lost. Hence, the lowest socially acceptable policy is pinned down by the preferences of the right-decisive judge. Similarly, the highest acceptable policy is determined by the left-decisive judge.

Example 2. *Consider a case $z = 0.45$, and suppose the court's disposition is $d = 1$. Consistency requires that $y \leq 0.45$. Suppose there are 6 judges in the majority, with ideal policies $x^1 = 0$, $x^2 = 0.2$, $x^3 = 0.25$, $x^4 = 0.3$, $x^5 = 0.4$ and $x^6 = 0.6$. Judges 1,..,5 (and presumably the 3 dissenting judges) cast sincere dispositional votes, whereas judge 6 voted strategically. Since policy-making requires a majority of the entire bench, $k = 5$. The left- and right-decisive judges, then, are judges 2 and 5, respectively. Suppose policy utility is given by $u_P(y, x) = -|y - x|$ and the common disagreement payoff is -1. Figure 1 depicts the set of socially acceptable policies for two values of δ.*

[10]This result usually follows from the concavity of players' preferences. In our model, preferences are not concave. Instead, the result is a consequence of the *IDID* property.

FIGURE 1. $A(\delta)$ **represents the social acceptance set given a cost of delay** δ**. When** $\delta = \frac{21}{26} \approx 0.8$**,** $A = [0.1, 0.45]$**, whilst if** $\delta = 0.95$**, then** $A = [0.2334, 0.3234]$**. Notice that when** $\delta \approx 0.8$**, the consistency constraint (i.e** $y \leq z$**) is binding. The dotted extension to** $A(\delta \approx 0.8)$ **represents the additional policies that would be socially acceptable absent the consistency constraint.**



In the first scenario ($\delta = \frac{21}{26}$), judges 2,3,4 and 5 are able to propose their ideal policies in equilibrium, whilst judge 1 must propose a compromise policy, i.e. the lowest policy acceptable to the right-decisive judge. It is infeasible for judge 6 to propose her ideal policy, and in equilibrium, she will propose the highest policy that is feasible $y = 0.45$. In fact, the left-decisive judge would in principle accept policies up to $y \approx 0.5$, however any policy above $y = 0.45$ would not rationalize the case disposition. In the second scenario ($\delta = 0.95$), judges 3 and 4 are able to propose their ideal policies in equilibrium, whilst the remaining judges must propose compromise policies. □

Example 2 demonstrates the essential features of the equilibrium. There are a range of policies that are potentially proposed in equilibrium. 'Moderate' judges may propose their ideal policies, whereas 'extreme' judges (in particular, judges who vote insincerely) must propose compromise rules. Whether a judge is 'moderate' or 'extreme' depends on the preferences and degree of patience of the left- and right-decisive judges. Moreover, the social acceptance set may be constrained by the facts of the case itself; the consistency requirement may be binding.

Comparative Statics on δ

The discount rate δ parameterizes the cost of delay in bargaining, or (equivalently) the relative 'importance' of the legal issue. As Example 2 demonstrates, it is also measures of the proposer's degree of agenda-control. When $\delta = 0$, delay is exceedingly costly relative to the importance to each judge of implementing desirable policies, so the non-proposing judges will accept any feasible policy. The proposer thus has complete control over the agenda and will propose the feasible policy closest to her ideal. Lemma 1 shows that, as $\delta \to 1$, the reverse becomes true; delay becomes costless relative to the importance of deciding the legal question correctly. The judges will bargain 'aggressively' over policy, such that, in equilibrium, the proposer loses control of the agenda entirely, and all judges will makes the same proposal. Thus, δ parameterizes the proposer's degree of agenda-control.

Lemma 1. *In any equilibrium,* $\overline{y}(\delta) > \underline{y}(\delta)$ *whenever* $\delta < 1$. *Moreover, there exists* μ *such that:* $\lim_{\delta \to 1} \overline{y}(\delta) = \mu = \lim_{\delta \to 1} \underline{y}(\delta)$.

Taken together, Proposition 1 and Lemma 1 make strong predictions about the size and composition of the 'join' and 'concur' coalitions. When delta is low, the cost of entertaining counter-proposals is sufficiently high that all judges will support the opinion of the court. The 'join' coalition will consist of all judges in the dispositional majority, and no judge will separately write a concurring opinion. By contrast, as $\delta \to 1$, judges become more demanding about the set of opinions which they will join. The size of the 'join' coalition will fall to a bare majority, consisting of either the left-most *or* right-most k judges. In either case, the 'concur' coalition will consist of judges from only one extreme (within the dispositional majority). Thus, regardless of the size of δ, an 'ends-against-the-middle' dynamic should never arise in which the 'join' coalition consists of relatively moderate judges, and extremists from both ends concur.

Limit Equilibria & 'Median Voter' Logic

Equilibrium policy-making by the Court is (generically) characterized by a menu of proposer-dependent policies. This feature arises whenever it costly for judges to make (or entertain) counter-proposals. Our approach, thus, stands in contrast to many existing studies that predict a unique policy outcome, typically by appealing to median-voter logic. However, as $\delta \to 1$, equilibrium in our model is also characterized by a unique policy that is proposed by all judges. Taking this limit, then, allows for fair comparisons between our model and those existing in the literature.

There is a tight connection between median-voter-type logic (or, more generally, the equilibrium concept of the core) and the limit equilibria of our bargaining game. For example, Cho and Duggan (2009) show that when agreement requires a simple majority, the limit equilibrium policy precisely coincides with the median agent's ideal policy. The intuition is straight-forward: the logic of the median-voter theorem is that whenever a policy is other than the median-voter's ideal is proposed, a majority coalition can be found that would replace it with something closer to the median-voter's ideal. This is true in our bargaining game as well, except that, when delay is costly, a non-core policy might persist, because it is too costly to make the counter-proposal that replaces it. As delay become costless, this friction disappears, and so the outcome of bargaining should coincide with the median-voter's ideal.

When agreement requires a super-majority, logic analogous to the median-voter theorem predicts an equilibrium outcome in the core.[11] However, under super-majority rule, the core generically contains many policies. In fact, the core is precisely the interval of policies bounded by the ideal policies of the left and right decisive voters. Whereas, under simple-majority rule, the median voter theorem identified a unique equilibrium policy, under super-majority rule, we have a continuum of

[11]The *core* is the set of policies for which there does not exist some other policy that is preferred to it by a winning coalition.

possible equilibrium policies. Parameswaran and Murray (2019) show that amongst this multiplicity, the limit equilibrium policy μ is focal – it is the one that is robust to making counter-proposals slightly costly. Thus, the bargaining limit can be thought of as a refinement that selects the most plausible core policy from amongst the multiplicity. (See Parameswaran and Murray (2019) for more details.)

An additional benefit of considering the limit equilibrium is that it admits a simple characterization. We briefly sketch a two-stage procedure for finding the limit policy. First, each judge in the dispositional majority joins one of two distinct factions, led by the left- and right-decisive judges. Second, the decisive judges determine policy by engaging in asymmetric Nash Bargaining, with weights proportional to the number of judges in their respective factions. As the weight on faction L increases, the resulting policy will move closer to the left decisive judge's ideal policy. Thus, for each judge, joining the left faction will cause the resulting policy to be further to the left than would have been the case had the judge joined the right faction. Applying this procedure gives the limit equilibrium policy provided that no judge would seek to switch factions after observing the resulting policy. (Intuitively, this requires that the factions be connected. If i joins faction L, so should all judges to her left, and vice versa.)

For example, suppose the judges separate into connected factions $\{1, ..., i\}$ and $\{i+1, ..., m\}$. Let $b_{i,i+1}$ denote the corresponding asymmetric Nash Bargaining solution[12]:

$$b_{i,i+1} = \arg\max_{y} \left[u_P(y, x^l) - u_P(D, x^l)\right]^i \cdot [u_P(y, x^r) - u_P(D, x^r)]^{m-i}$$

The limit equilibrium policy is $b_{i,i+1}$ provided that ideal policies of judges $1, ..., i$ are to the left of $b_{i,i+1}$ and the ideal policies of judges $i+1, ..., m$ are to the right.

In many circumstance, such a clean separation into consistent factions is possible, and the above characterization holds. However, in other instances, a problem arises: Consider some 'moderate'

[12]Our notation emphasizes that the factions consist of judges $\{1, ..., i\}$ and $\{i+1, ..., m\}$.

judge i and suppose that all judges $1, \ldots, i-1$ join the left faction and all judges $i+1, \ldots, m$ join the right faction. It may be that if i joins the left faction, the resulting policy will move further to the left than i's ideal. If so, judge i would want to switch to the right faction. But joining the right faction may cause the policy to move further to right than i's ideal policy, and so judge i would want to switch back to the left faction. Judge i is *pivotal*. There is no consistent way for the limit policy to be either to her left or right; the only possibility is that it coincides with her ideal. It is as if the pivotal judge 'straddles' the two factions.

Parameswaran and Murray (2019) show that the limit equilibrium policy μ of the bargaining game is characterized by one of these two possibilities. Either it is the Nash bargaining solution when the judges separate into consistent factions, or it is the ideal policy of some pivotal judge. We have:

Proposition 2. *Let* $i^* = \min\{i \,|x_i > b_{i,i+1}\}$. *Then:* $\mu = \min\{x_i^*, b_{i^*-1,i^*}\}$

We illustrate Proposition 2 in the following example:

Example 3. *Suppose* $m = k = 5$*, so that the dispositional majority is a bare majority of the Court. Then* $l = 1$ *and* $r = 5$*. Suppose policy preferences are bell-curve shaped* $u_P(y, x) = e^{-\frac{1}{2}(y-x)^2} - 1$*, the disagreement payoff is* $u_P(D, x) = -1$*, and normalize:* $0 = x_1 \leq x_2 \leq \ldots \leq x_5 = 0.5$*. The Nash bargaining solution when the left and right factions are* $\{1\}$ *and* $\{2, 3, 4, 5\}$ *is* $b_{1,2} = 0.4$*. Similarly, for the remaining cases, we have:* $b_{2,3} = 0.3$*,* $b_{3,4} = 0.2$*, and* $b_{4,5} = 0.1$*. Then:*

$$\mu = \begin{cases} b_{1,2} = 0.4 & x^2 > 0.4 \\ x^2 & 0.3 \leq x^2 \leq 0.4 \\ b_{2,3} = 0.3 & x^2 < 0.3 < x^3 \\ x^3 & 0.2 \leq x^3 \leq 0.3 \\ b_{3,4} = 0.2 & x^3 < 0.2 < x^4 \\ x^4 & 0.1 \leq x^4 \leq 0.2 \\ b_{4,5} = 0.1 & x^4 < 0.1 \end{cases}$$

As a check on the logic of Proposition 2, suppose the judges divide into factions $\{1, 2, 3\}$ *and* $\{4, 5\}$. *The resulting asymmetric Nash bargaining solution is* $b_{3,4} = 0.2$. *The conjectured factions are consistent with this policy provided that* $x_3 < 0.2 < x_4$, *as the example states. We can verify that, under that alignment of preferences, any other composition of factions will induce policies that are inconsistent, in the sense that at least one judge would want to switch factions. By contrast, suppose* $x_4 = 0.18$. *If judge* 4 *joined the left faction, the induced policy would be at least as low as* $b_{4,5} = 0.1$, *which is further to the left than judge* 4 *would tolerate; she would want to switch to the right faction. By contrast if she joined the right faction, the induced policy would be at least as high as* $b_{3,4} = 0.2$ *which is further to the right than she would tolerate; she would wish to switch to the left faction. When* $x_4 = 0.18$, *judge* 4 *is pivotal.* □

We note some features of the equilibrium mapping. First, for each judge between the decisive judges, there is some arrangement of ideal policies for which they are pivotal. In Example 3, the left- and right-decisive judges were judges 1 and 5. Then, as $\delta \longrightarrow 1$, the equilibrium policy potentially reflects the ideal policies of any of judges 2, 3 and 4. In particular, the median judge in the majority (judge 3) is not generically privileged. Additionally, there are arrangements of ideal policies under which no judge is pivotal, and the equilibrium policy is the solution to the asymmetric Nash Bargaining problem between the decisive judges.

Second, bargaining pushes the equilibrium policy towards the 'middle' of the core. In Example 3, the core is the interval $[0, 0.5]$. When the ideal policy of judge 3 (the majority-median) is in the middle of this interval (i.e. $x^3 \in [0.2, 0.3]$), then the median of the majority is indeed pivotal. However, as the median's ideal policy becomes extreme, the equilibrium switches to some other less extreme policy. For example, if $x^3 > 0.3$, so that the median is further to the right, then equilibrium policy switches to a policy below the median's ideal. Initially it switches to $b_{2,3}$ —the policy that results from the judges dividing into factions $\{1, 2\}$ and $\{3, 4, 5\}$. However, this policy will cease to be equilibrium consistent if judge 2's ideal policy shifts too far to the right (i.e. if $x^2 > 0.3$). If so, then the equilibrium switches to judge 2's ideal policy, and if this too becomes extreme (i.e. if

$x^2 > 0.4$), then the equilibrium shifts to the Nash bargaining solution associated with blocs $\{1\}$ and $\{2, .., 5\}$. Hence, bargaining exerts a moderating force that keeps the equilibrium closer to the middle of the core than would be the case under the median voter theorem.

In strong contrast to existing results, our analysis shows that the equilibrium policy will generically not coincide with either the median judge on the bench[13], nor the median judge in the dispositional majority. This should not be surprising. The logic of the median voter theorem is particular to decision making under simple-majority rule. But, since policy-making by the court often proceeds under an effective super-majority rule, there is no reason to privilege the median judge over the others.

In this paper, we do not take up the issue of nominations to the bench. However, we briefly note the stark implications of Proposition 2 for the president's optimal nomination's choice. Importantly, equilibrium outcomes depend not only on the relative ordering of the judges' ideal policies, but their absolute location in policy space. The president's nomination problem, thus, is not simply a 'move-the-median' game. The president could nominate two different judges, both occupying the same relative position in the ordering, but with different implications for the equilibrium policies chosen.

THE ADJUDICATION STAGE

First Round Assignment

In the first stage, each judge must cast a dispositional vote, taking into account the equilibrium policies that will result, given differently composed majority coalitions. This policy, in turn, depends on which judge is selected by the most senior judge in the majority (often the chief justice) to draft

[13]We establish in section 4 that the policy will coincide with the ideal policy of the median judge *if* when the dispositional vote is unanimous.

the initial proposal. For each majority coalition $M \subset \{1, ..., n\}$, let $s(M, d, z) \in M$ denote the judge who is selected to make the first proposal. Additionally, let $\gamma(M, d, z) = y^{s(M,d,z)}$ be the policy that the selected judge will propose in equilibrium.

The function s depends on the particular incentives faced by the chief judge. We might naively suppose that the chief is purely motivated to maximize her utility from the case. But this would imply that the chief judge always assigns the opinion to herself – an implication at odds with the actual practice of recent chiefs. Indeed, the court maintains a practice of sharing the workload of opinion writing amongst its members. This can rationalized by noting that opinion writing is costly, and so the chief makes her assignment choice taking into account the associated direct and opportunity costs. Given the many additional incentives that would need to be incorporated, it is clear that providing micro-foundations for the chief's selection is outside the scope of this paper.

Instead, we take a reduced-form approach, taking the selection function s as given. We assume s and γ satisfies the following:

Assumption 1. *Let $M, M' \subset \{1, ..., n\}$ be majority coalitions.*

1. *Suppose $j \notin M$. Then $u_P(\gamma(M \cup \{j\}), x^j) \geq u_P(\gamma(M), x^j)$.*
2. *Suppose for every $i \in M$, there exists $j \in M'$, such that $y^i(z, M, \delta) = y^j(z, M', \delta)$. Then $\gamma(M) = \gamma(M')$.*

Assumption 1 is in two parts. The first part states that when a new member joins the coalition, the chief should not respond in a way that makes the resulting policy *worse* from the new judge's perspective. The assumption is akin to the independence of irrelevant alternatives. By joining the majority coalition, a new judge may cause the resulting opinion to be closer to her ideal, for example, if the chief recognizes her to author the opinion. However, the assumption disallows the chief's assignment to causes policy to move in the opposite direction.[14]

[14]Indeed, part 1 of Assumption 1 is a direct consequence of the independence of irrelevant alternatives whenever δ is not too small.

The second part states that, when confronted with two different coalitions that induce the same set of policy proposals, the chief should not make selections that cause different policies to be induced in the different instances. If replacing judge i in the coalition with judge j does not change the set of equilibrium proposals, the chief should treat judges i and j as perfect substitutes. The outcome induced when one is included in the coalition should be identical to the outcome when only the other is included.

Taken together, the two parts of Assumption 1 are intended to capture, in reduced form, structurally sound decision-making by the chief. (Of course, as $\delta \to 1$, the chief's selection becomes inconsequential, as all judges in a given coalition will propose the same policy.)

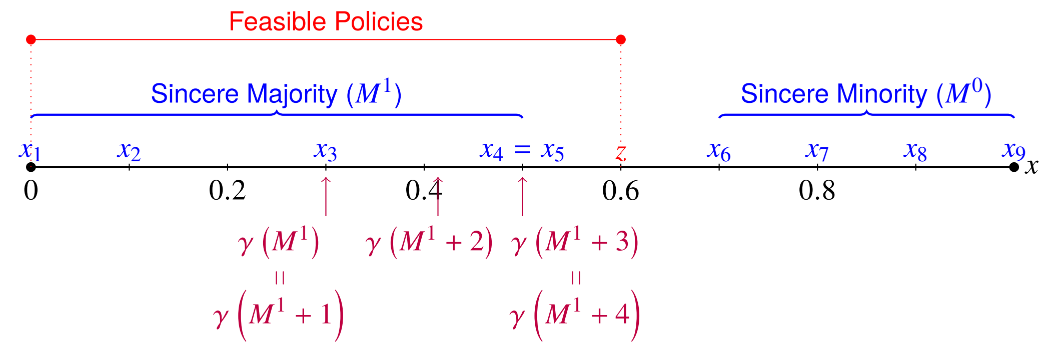
Optimal Dispositional Coalitions

Fix a case z. Let $M^0(z)$ and $M^1(z)$ denote the sets of judges who would, if voting sincerely, choose dispositions '0' and '1', respectively (i.e. $M^0(z) = \{i \mid z < x^i\}$ and $M^1(z) = \{i \mid z > x^i\}$). Let (d^*, M^*) denote an adjudication equilibrium, where $d^* \in \{0, 1\}$ denotes the disposition of the court, and M^* denotes the equilibrium majority coalition.

Lemma 2. *Every judge who sincerely agrees with the equilibrium disposition of the court will join the majority coalition. Formally, if (d^*, M^*) is an adjudication (Nash) equilibrium, then $M^{d^*}(z) \subset M^*$.*

For a given equilibrium disposition d^*, Lemma 2 states that all judges who sincerely agree with the disposition of the case will be in the majority coalition. The intuition is straight-forward: Being in the majority coalition is always beneficial on the policy-utility dimension in that it enables a judge to influence the equilibrium policy of the court, and pull the policy (weakly) closer to her ideal. Furthermore, if the judge votes sincerely, she does not suffer a loss on the expressive dimension. When a judge agrees with the court's disposition, her expressive and policy motives are not in conflict. It is a dominant strategy for all such judges to vote sincerely.

FIGURE 2. Equilibrium policies for differently sized dispositional coalitions.



Judges who disagree with the disposition of the court face a genuine trade-off. Voting strategically enables them to influence the equilibrium proposal, but incurs the expressive cost of voting insincerely. As we will see, the policy benefit of voting strategically (for each judge) depends on whether (and how many) other judges are also voting strategically. This gives rise to possibly many Nash equilibria in the adjudication game, as the following example demonstrates:

Example 4. *Let* $z = 0.6$*. Suppose policy utility is bell-curve shaped:* $u_P(y, x) = e^{-\frac{1}{2}(z-x)^2} - 1$*, and the vector of ideal policies is:* $(x^1, ..., x^9) = (0, 0.1, 0.3, 0.5, 0.5, 0.7, 0.8, 0.9, 1)$*. The disagreement payoff is* $u_P(D, x) = -1$*, and* $\delta \to 1$ *so that all judges make the same proposal. (See Figure 2.)*

Suppose the equilibrium disposition is $d^* = 1$*. By Lemma 2, judges 1-5 will definitely be in the majority. The equilibrium policies are* $\gamma(M^1) = 0.3 = \gamma(M^1 + 1)$*,* $\gamma(M^1 + 2) \approx 0.414$*, and* $\gamma(M^1 + 3) = 0.5 = \gamma(M^1 + 4)$*, where* $\gamma(M^1 + p)$ *is the equilibrium offer when the majority coalition consists of judges 1-5 (i.e.* M^1*) and any* $p \in \{1, 2, 3, 4\}$ *of the remaining judges. (Judges* $6, ..., 9$ *are perfect substitutes; outcomes do not depend on which of the judges vote strategically —just how many.) The adjudication Nash equilibria (in which* $d^* = 1$*) are given Table 2:*

TABLE 2. Equilibrium coalitions that implement disposition $d^* = 1$. Equilibria with sincere voting (i.e. the dispositional coalition is M^1) always exist. For $\alpha < 0.273$, there are also equilibria in which judges in the sincere minority vote strategically. For $\alpha < 0.117$, there can be multiple equilibria with strategic voting.

Salience	$\alpha < 0.117$	$\alpha \in (0.117, 0.129)$	$\alpha \in (0.129, 0.273)$	$\alpha > 0.273$
# Equilibria	5	2	2	1
Sincere	M^1	M^1	M^1	M^1
Strategic	$M^1 \cup \{6, 7, 8\}$ $M^1 \cup \{6, 7, 9\}$ $M^1 \cup \{6, 8, 9\}$ $M^1 \cup \{7, 8, 9\}$	$M^1 \cup \{6, 7, 8\}$	$M^1 \cup \{6, 7\}$	None

□

Unlike the policy sub-game, where the equilibrium was unique, the strategic incentives at the adjudication stage result in there being potentially many adjudication (Nash) equilibria. There are two sources of multiplicity, both apparent in Example 4, and both having to do with coordination. For ease of exposition, suppose $\alpha < 0.117$, so that strategic voting is costly, but this cost is small relative to the resulting policy gains.

First, note that it is an equilibrium for any three of the four judges in the sincere minority to vote strategically. (There is no policy benefit to the fourth judge from voting strategically, and there is a policy cost to any of the three judges who voted strategically to switch to voting sincerely instead.) Since it does not matter which judges vote strategically and which one votes sincerely, there are four such equilibria. In this context, strategic voting by the judges exhibits 'strategic substitutability'. The judges in the sincere minority are effectively playing a game of chicken — they face a coordination problem in deciding which judges will vote strategically and which will vote sincerely.

Second, it is also an equilibrium for all judges to vote sincerely. (There is no policy benefit to having one judge from the sincere minority vote strategically, since doing so does not shift the policy.)

Hence, strategic voting by the judges exhibits 'strategic complementarity'. No judge would vote strategically if none of the others do. However, if at least one other judge voted strategically, then each of the remaining judges has an incentive to vote strategically as well. Starting from the sincere coalition, it takes a joint deviation by a group of judges to make strategic voting attractive.

The example also illustrates how the composition of dispositional coalitions responds to changes in the salience of expressive utility. As α increases, so does the cost of voting strategically, and so the possibility of sustaining various equilibria with strategic voting decreases. By the single-crossing property, the expressive cost of insincerity is higher for judges with more extreme ideal policies. Thus, as α increases, judge 9 is the first to cease voting strategically, then judge 8, etc.

Given the presence of multiple equilibria, we seek to focus attention on the equilibrium that is most plausible. To identify this 'focal' equilibrium, we use two refinement criteria, limiting attention to adjudication equilibria that are *connected* and *coalition proof*.

An adjudication equilibrium is *connected* if both the majority and minority dispositional coalitions are connected. If a coalition is disconnected, then it must be that a relatively moderate judge voted sincerely whilst a relatively extreme judge voted strategically. However, given the single-crossing property, strategic voting is more costly for relatively extreme judges than moderate ones. Thus, it is more reasonable to expect moderate judges to vote strategically than extreme judges. It is also more 'likely', in the sense that strategic voting by the moderate judge can be sustained in equilibrium over a larger range of values of α than strategic voting by an extreme judge (as seen in Example 4). Thus, hen there are multiple adjudication equilibria resulting from strategic substitutes, connectedness selects the one that is most plausible.

An adjudication equilibrium is *coalition proof* if, not only would no individual judge benefit from deviating unilaterally, but no stable group of judges could mutually benefit by a joint deviation[15] (see Bernheim et al. (1987)). The refinement rules out equilibria in which a subset of agents are

[15] Stability requires that when considering group deviations, we limit attention to groups for which there does not exist a subgroup who could profitably deviate from the deviation.

trapped in a situation that is inferior, but from which they could jointly and stably escape. On collegial courts, it is not unreasonable to assume that communication between the judges can enable a coalition of judges to jointly affect a favorable deviation. When strategic complementarity creates multiple equilibria, coalition-proofness selects the equilibrium that is 'most plausible', in the sense of ensuring that those complementarities are exploited as far as possible. We show in Lemma 4 in the Appendix that the refinement selects the adjudication equilibrium with the 'largest' coalition, guaranteeing that all complementarities are fully exhausted. A consequence is that equilibrium dispositional majorities exhibit greater (apparent) cohesion than would be the case if all judges voted sincerely.

We can apply the notions of connectedness and coalition-proofness to select a focal equilibrium from the multiplicity in Example 4 when $\alpha < 0.273$. (When $\alpha > 0.273$, there is a unique adjudication equilibrium, and so there is no selection to make.) First, note that when $\alpha < 0.117$, connectedness rules out three of the four equilibria with strategic voting — the ones the require judge 9 to vote strategically. This is reasonable since judge 9 is most extreme, and thus finds it most costly to vote strategically. Two candidate equilibria remain; the sincere equilibrium and the equilibrium in which judges 6, 7 (and for some values of α, judge 8) vote strategically. Coalition-proofness selects the equilibrium with the larger majority coalition, consistent with strategic voting.

We are now ready to characterize the main results in this section.

Proposition 3. *There exists a Connected Coalition-Proof Adjudcation Equilibrium (CCPAE). Moreover, in any CCPAE* (d, M)*:*

- *If* $d = 1$*, then* $M = \{1, ..., j_1\}$*, where* $j_1 \geq \frac{n+1}{2}$.
- *If* $d = 0$*, then* $M = \{j_0, ...n\}$*, where* $j_0 \leq \frac{n+1}{2}$.

Proposition 3 shows that a CCPAE always exists; i.e. there is (at least) one that Nash equilibrium that survives the refinements that we impose. Proposition 3 also describes the features of equilibrium coalitions. In any CCPAE, the majority coalition will contain all but (possibly) the most extreme-right judges, if the disposition is '1', or all but (possibly) the most extreme-left judges, if the disposition is '0'. An immediate implication is that the median judge will always be in the dispositional majority and so the median justice is 'pivotal' over the case disposition. We stress that whilst the median justice is pivotal, the equilibrium disposition need not coincide with the median judge's sincere assessment of the case; she may vote strategically (see Example 5).

A related implication of Proposition 3 is that the median judge will always be one of the decisive judges in the policy-making stage. However, unless the dispositional vote is unanimous, some other judge will also be decisive. To the extent that opinion-writers have agenda-setting power, the median judge may still be able to implement her ideal policy if she is assigned to write the opinion. However, as this agenda-setting privilege disappears (i.e. as $\delta \to 1$), the median judge's ideal policy will generically *not* be implemented. Instead, the equilibrium policy will either be to his left or right, depending on whether the majority coalition contains mostly leftist or rightist judges.

Whilst Proposition 3 guarantees that a CCPAE exists, it doesn't guarantee that the CCPAE is unique.

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Corollary 1. *There exist at most two CCPAE.*

1. *If (d, M) and (d', M') are distinct CCPAE, then $d \neq d'$.*
2. *For each $z \in [0, 1]$, there exists $\alpha(z) \geq 0$, such that if $\alpha > \alpha(z)$, the CCPAE is unique.*

Corollary 1 tells us that there are no more than two CCPAE. Part 1 tells us that if there are two CCPAE, one will be associated with disposition $d = 0$ and the other with disposition $d = 1$. Hence, our refinements isolate the focal equilibrium associated with each of the two possible dispositional outcomes.

Part 2 tells us that, as expressive preferences become sufficiently salient, there can only be one CCPAE. When the expressive cost of insincerity is large, sufficiently many judges will vote sincerely,

TABLE 3. CCPAE Equilibria in Example 5

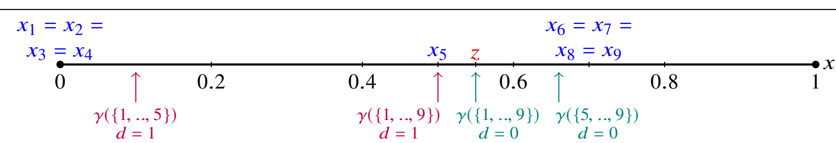
Salience	$\alpha < 0.118$	$\alpha \in (0.118, 0.585)$	$\alpha \in (0.585, 1.285)$	$\alpha > 1.285$
No. CCPAE	2	2	1	1
$d = 1$	$M = \{1, \ldots, 9\}$	$M = \{1, \ldots, 9\}$	None	$M = \{1, \ldots, 5\}$
$d = 0$	$M = \{1, \ldots, 9\}$	$M = \{5, \ldots, 9\}$	$M = \{5, \ldots, 9\}$	None
Potentially Strategic?	All	$\{5, \ldots, 9\}$	$\{5\}$	None

and this prevents one of the dispositional outcomes from arising in equilibrium. (In many scenarios, but not always, the prevailing outcome will be the one that would arise if all judges cast their dispositional votes sincerely.) By contrast, when α is low enough, policy preferences dominate the decision-making of enough judges, that both outcomes can be sustained as equilibria. Judges are strongly motivated to be in the dispositional majority so that they can affect policy-making. The adjudication game resembles a dispositional coordination game. Most judges care less about which disposition prevails than ensuring that they are part of the majority coalition. In particular, there will be equilibria in which a strong majority of judges favor one disposition, and yet the other disposition is chosen.

The following example illustrates each of the points discussed above:

Example 5. *Consider a case* $z = 0.55$. *Suppose that policy preferences are bell-curve shaped:* $u_P(y, x) = e^{-\frac{1}{2}(y-x)^2} - 1$, *the disagreement payoff be* $u_P(D, x) = -1$, *and* $\delta \to 1$. *Let the ideal policies be:* $x^1 = x^2 = x^3 = x^4 = 0 < x^5 = 0.5 < 0.7 = x^6 = x^7 = x^8 = x^9$ *(i.e. there is a relatively extreme homogeneous left bloc of 4 judges, a relatively moderate right bloc of 4 judges, and a centrist median judge). The median judge's ideal disposition is* $d^* = 1$. *Figure 3 illustrates this setup, and the equilibrium policies that will result, for each dispositional outcome, depending on whether the minority bloc votes strategically or not. The CCPAE are described in Table 3.*

FIGURE 3. Equilibrium policies chosen for differently composed dispositional majorities.



There is a unique equilibrium whenever $\alpha > 0.5848 = \alpha(z)$. *Even when equilibria are unique (i.e the median judge is pivotal), the outcome need not coincide with her ideal disposition. There will be strategic voting unless* $\alpha > 1.2848$. *Moreover, as* α *increases, more extreme judges become less likely to vote strategically. Thus, the median judge potentially votes strategically over the largest range of* α*, whereas the left bloc of judges vote strategically over the smallest range.* □

As Example 5 illustrates, when α is low, regardless of their actual preferences, there is a CCPAE in which all judges choose disposition $d = 1$, and a CCPAE where they all choose disposition $d = 0$. A similar result arises in Fischman (2008), although the mechanism is different. In Fischman's model, unanimity arises because it is costly to dissent (for example, because it would require the judge to expend resources writing a dissenting opinion). In our model, unanimity arises because the hedonic cost of voting insincerely is low relative to the policy gains.

Comparative Statics

Effect of Salience of Expressive Utility Example 5 showed that the incentives for judges to vote strategically varied with the salience of expressive preferences α, and the distance of the case from each judge's ideal threshold. Intuitively, as expressive concerns become more salient, strategic voting becomes harder to sustain, and so the majority coalition shrinks in size. If expressive concerns are sufficiently large, then no judge will vote strategically, and equilibrium coalitions and case dispositions will reflect the sincere preferences of the judges.

Fix a case z. Let $d(z) = \mathbf{1}[|M^1(z)| > |M^0(z)|]$ denote the sincere disposition, which is the disposition that would prevail if all judges voted sincerely. Similarly, let $M(z)$ denote the sincere majority coalition: $M(z) = M^{d(z)}(z)$. The above ideas are reflected in Lemma 3, and are illustrated in Example 5 and Figure 4, below.

Lemma 3. *The following are true:*

1. *The size of equilibrium coalitions (with the same disposition) is decreasing in expressive concerns. (Formally, let (d, M) and (d', M') be CCPAE associated with salience levels α and α', with $\alpha > \alpha'$. If $d = d'$, then $M \subseteq M'$.)*
2. *When there are no expressive concerns, the Court's decisions will be unanimous. (If $\alpha = 0$, then $M = \{1, .., n\}$.)*
3. *When expressive concerns are sufficiently large, the unique CCPAE is characterized by sincere voting. (For a given case z, there exists $\overline{\alpha}(z) > 0$ s.t. $\forall\, \alpha > \overline{\alpha}(z)$ there is a unique CCPAE (d, M) with $d = d(z)$ and $M = M(z)$.)*

Part 2 of the Lemma merits further comment. It states that the court will be unanimous in any CCPAE when expressive concerns are not salient. In practice on the Supreme Court, dissents by at least one judge are common, and 5-4 dispositional votes are not uncommon. Thus, we highlight the important role that expressive preferences play in describing behavior on the Court. Neither our model, nor any that is broadly similar, would be able to explain dissents if limited to judicial preferences that were purely consequentialist.[16]

Effect of Case Location A key insight of this paper is that rule-making by courts cannot be divorced from the specific facts of the case being adjudicated. (This stands in contrast to 'legislature-like' models of the judiciary, where the court purely focuses on policy-making, to which end the facts of the instant case are incidental.) Example 2 demonstrated that the case facts directly affected the set of feasible policies that the Court could implement, and that the dispositional consistency requirement might be binding.

[16]A model in search of a consequentialist account would necessarily be dynamic; the role of the dissent is to increase the likelihood of the current policy being over-turned in the future. Whilst we do not deny the merits of such an argument, we do note the many complexities such a model invites. For example, in any such model, judges will be unable to commit to implement currently chosen policies in the future. This would significantly dampen the import of policy-making today, and thus diminish the value of the dissent.

Case location also affects the composition of the dispositional majority, and this will likely affect the equilibrium policy, even when the consistency constraint is non-binding. This occurs for two reasons. First, suppose all judges cast dispositional votes sincerely. Then, starting from the median judge's ideal threshold, as cases becomes more extreme, the number of judges who find themselves in the majority will (rather obviously) increase.

Second, and more subtly, changing the case location can change the incentives for judges to vote strategically, and thus affect the composition of the dispositional majority. To see this, consider the example below, which considers two cases that would both result in the same dispositional majority if judges voted sincerely:

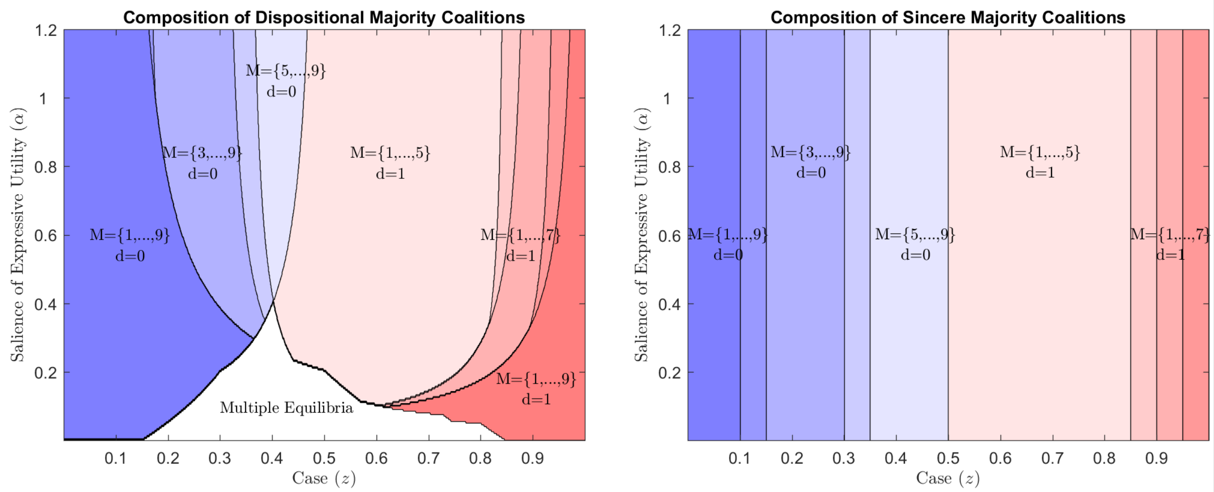
Example 6. *Suppose that policy preferences and ideal policies are as in Example 5. Let* $\alpha = 0.3$*. Consider two cases:* $z_1 = 0.1$ *and* $z_2 = 0.4$*. In both scenarios, the case is located between the ideal policy of the left bloc and the median judge, so that the sincere disposition and sincere majority coalition would be* $d(z_i) = 0$ *and* $M(z_i) = \{5, .., 9\}$*. Both scenarios admit a unique CCPAE, with disposition* $d = 0$*.*

- *When* $z = 0.1$*, the majority coalition will be the entire bench* $M = \{1, .., 9\}$*, and the equilibrium policy will be* $\gamma = 0.5$*. There is strategic voting by the left bloc.*
- *When* $z = 0.4$*, the majority coalition will consist of a bare majority* $M = \{5, .., 9\}$*, and the equilibrium policy will be* $\gamma = 0.66$*. There is no strategic voting.*

In both scenarios, each judge would ideally decide the cases the same way. However, when z *is close to the left bloc's threshold, the cost of strategic voting is lower, and thus the judges are more inclined to vote strategically, to pull the ideal policy closer to their ideal.* □

Although the set-up in Example 6 is stark, it reflects a more general relationship between case location, the composition of dispositional majorities, and the policy of the court. We see this general relationship in Figure 4 below:

FIGURE 4. Impact of case location and the salience of the expressive utility on the composition of the dispositional majority, and the resulting policy. Policy preferences are bell-curve shaped: $u_P(y,x) = e^{-\frac{1}{2}(z-x)^2} - 1$, the disagreement payoff is $u_P(D,x) = -1$, and $\delta \to 1$. The vector of ideal policies be $(x^1, ..., x^9) = (0.1.0.15, 0.3, 0.35, 0.5, 0.85, 0.9, 0.95, 1)$. The left panel shows actual CCPAE dispositions and majority coalitions. The right panel shows dispositions and coalitions if the judges voted sincerely.



The left panel of Figure 4 shows how the equilibrium dispositions and coalitions vary as a function of case location and the salience of expressive utility. The blue and red areas represent regions where the majority disposition is $d = 1$, and $d = 0$, respectively. Darker regions indicate larger coalitions. The right panel represents the disposition and majority coalitions if judges voted sincerely. These regions should be vertical bands, since the outcome under sincere voting is independent of α. Since the boundaries are vertical when judges vote sincerely, one way to observe the extent of strategic voting is to see how 'sloped' or 'curved' the boundaries of the regions are.

The results from Lemma 3 are also evident in Figure 4. First, fixing a case, z, as α increases, the number of judges in the majority coalition decreases. The extent of strategic voting decreases as the salience of expressive utility increases. Second, as α becomes sufficiently large, the lines become vertical; for α large enough, the equilibrium coalitions coincide with the coalitions that would arise under strategic voting. Finally, fixing any α, we notice that as the case becomes more extreme, equilibrium coalitions are more likely to be larger, and the likelihood of strategic voting increases. Indeed, since $x^9 = 1$, if judges voted sincerely, judge 9 would always choose $d^9 = 0$. However, allowing for strategic voting, judge 9 potentially chooses $d = 1$ over a large range of cases, when α is low.

Tying the results from sections 3 and 4 together, then, yields the following insight. When the case is ‘moderate’ (in the sense of being close to the median judge’s ideal threshold), then majority coalitions are likely to be smaller, and the resulting equilibrium policy is likely to be more extreme (in the sense of being farther from the median judge’s ideal). As the case becomes more ‘extreme’ (i.e. farther from the median judge’s threshold), then majority coalitions will become larger, and the resulting policy will likely be more moderate (i.e. closer to the median judge’s ideal).

DISCUSSION

The model opens new avenues for studying institutions that jointly produce judgments and policies. To illustrate, we briefly discuss 9 empirical implications about decision-making on the U.S. Supreme Court.

Strategic Dispositional Voting

Express indications of strategic voting are understandably rare among the justices. However, there are exceptions. For example, in *Arizona v. Fulminante*, the Supreme Court reviewed a criminal conviction that rested on the defendant’s confession. The dispositional vote hinged on whether the defendant’s confession was coerced. Then, contingent on a coerced confession, the dispositional majority addressed policy questions about the exclusionary rule. In his opinion on the majority side, Justice Kennedy indicated that he believed the confession was voluntary, hence, joining the dissenters would have been the sincere dispositional vote. Of course, a strategic one allowed him to address the policy questions at hand. Indeed, Kennedy cast the decisive dispositional vote in the case.

A small empirical literature examines strategic dispositional voting on the U.S. Supreme Court (Arrington and Brenner (2004); Johnson et al. (2005); Clark et al. (2018)). In addition, studies of the Chief Justice often note his incentive to vote strategically on the disposition, to control the initial opinion assignment (c.f. Danelski and Ward (2016), Maltzman and Wahlbeck (1996)). However, the absence of a strong theory of strategic dispositional voting has limited these studies.[17] The sequential bargaining model points to a new approach, by distinguishing likely-sincere votes from possibly-strategic ones. Contrasting the voting patterns for the same justice using the two types of votes becomes the critical comparison. More specifically:

1. The model predicts that dissents are always sincere, as are join versus concur decisions, while votes to join the dispositional majority may be strategic. Thus, the required test simply compares voting patterns by justice using votes sorted this way.
2. The model predicts that dispositional voting in high-α cases (those in which the disposition is critical while policy is not) is likely sincere while voting in low-α cases (where dispositions matter little but policy is extremely important) may not be. *Bush v. Gore*, settling a recount dispute in Florida's 2000 presidential election, illustrates a likely high-α case. More systematically, many death penalty cases seem disposition-critical rather than policy-critical. Given the identification of sufficient number of high- and low-α cases, the required contrast in voting patterns by justice can be implemented.
3. The model predicts that strategic dispositional voting is less likely for justices located on the ideological wings of the Court than those located in the center of the Court. Some justices have served for extended periods during which they moved from wing to center or vice versa due to exogenous changes in the Court's make-up. The resulting natural experiment offers an attractive venue for implementing the required contrast in decisions to join the dispositional majority.

[17]For example, if the Court's median justice determines Court policy, a strategic dispositional vote is pointless since the Court's policy will be the same irrespective of the case disposition.

Policy Coalitions

In the absence of theories explaining why policy coalitions matter, scholars have devoted little attention to them.[18] But because policy coalitions —defined by joins versus concurs in the dispositional majority —are easy to observe they offer an attractive venue for testing models that actually make predictions about policy coalitions.[19] Our model offers the following testable predictions about policy coalitions.

4. Join coalitions will be connected, and built from one ideological side of the dispositional majority or the other, but not from the center toward both wings. In other words, few join coalitions will feature "both ends against the middle" join/concur voting.
5. In cases where the bargaining intensity is very high (i.e., the parameter δ approaches 1) the size of the join coalition will approach 5.
6. By contrast, when bargaining intensity is very low (i.e., the parameter δ approaches 0), concurs will disappear. Thus, the join coalition will tend to include the entire dispositional majority.

Policy Content of Majority Opinions

The model makes many distinctive predictions about the policy content (spatial location) of majority opinions. Among the more striking are:

[18] Models that equate majority opinion content with the ideal point of the median justice on the Court leave no room for policy coalitions to matter. Strict monopoly author models also deprecate policy coalitions, since only the identity of the opinion author matters for opinion content.

[19] Dissents have received considerable empirical study but concurrences and joins much less; Corley (2010) and Hettinger et al. (2003) are exceptions. This literature has tended to see concurrences as reflecting a breakdown in a norm of consensus rather than the content (spatial location) of the majority opinion. However, using the papers of the justices, Epstein and Knight (1997) provide insight into bargaining over majority opinion content, including statements about joining, concurring, and dissenting.

7. Fixing the identity of the members of the majority dispositional coalition, there will be a distribution of majority opinion locations associated with the coalition, not a single location.
8. When bargaining intensity is high (i.e., δ approaches 1), the majority opinion locations will be drawn to the "center" of the dispositional majority, where that center is measured by the weighted Nash Bargaining solution (a spatial location that can readily be calculated).
9. When bargaining intensity is low, (i.e, δ approaches 0) majority opinions will move closer to the ideal point of the opinion author, in other words, author influence will be greater.

These hypotheses await a solution to the tough measurement problem of locating majority opinions in the same space as justice ideal points. However, recent advances in scaling suggest that it may soon be feasible to test these (and other) hypotheses about majority opinion locations (see for instance Clark and Lauderdale (2010)).

CONCLUSION

In this paper, we present a new model of decision-making on multi-member bodies that simultaneously dispose of cases and formulate a rule that rationalizes the case disposition. We focus on one procedure, the "American procedure", for joint production of judgments and policies. Although apex courts outside the U.S. use somewhat different procedures, the analytic methods developed here can be modified to study those institutions as well.

We show that the American procedure leads to profound interactions between judgments and policies. As a result, treating the U.S. Supreme Court (for example) as a "little legislature" is problematic because it ignores how judgments tether policy outcomes. Similarly, treating the U.S. Supreme Court as a "big jury" may misrepresent judgments to a degree, because it ignores the possibility of strategic dispositional voting aimed at shaping subsequent policymaking. The model makes specific predictions about dispositional coalitions, policy coalitions, and the content (spatial location) of majority opinions. These offer new directions for the empirical study of apex appellate courts and regulatory bodies.

There are obvious possibilities for further theoretical development. A valuable one would be case selection, because case locations emerge as critical for both judgments and policy. A second extension involves *stare decisis* and the time consistency of policymaking (Baker and Mezzetti (2012)). For example, when will the Court obey its own precedents, and when might it deviate from them? (Cameron et al. (2019)) Extending the joint production framework to different procedures, especially those used in non-American constitutional and international courts, would be a valuable departure in comparative institutional analysis. Additionally, as with most formal models, our framework assumes that the judges' preferences are common knowledge. In practice, some preference uncertainty may exist —although on Collegial courts, we think this concern will be more muted than in other settings —and this will naturally affect the equilibrium behavior of judges at both the policy and adjudication stage. We leave the analysis of behavior under asymmetric information of this sort to future studies.

REFERENCES

Arrington, Theodore S and Saul Brenner (2004). Strategic voting for damage control on the supreme court. *Political Research Quarterly 57*(4), 565–573.

Ash, Elliott and W Bentley MacLeod (2015). Intrinsic motivation in public service: Theory and evidence from state supreme courts. *The Journal of Law and Economics 58*(4), 863–913.

Baker, Scott and Claudio Mezzetti (2012). A theory of rational jurisprudence. *Journal of Political Economy 120*(3), 513–551.

Banks, Jeffrey S. and John Duggan (2000). A bargaining model of collective choice. *The American Political Science Review 94*(1), pp. 73–88.

Banks, Jeffrey S and John Duggan (2006). A general bargaining model of legislative policy-making. *Quarterly Journal of Political Science 1*(1), 49–85.

Baron, David P (1996). A dynamic theory of collective goods programs. *American Political Science Review 90*(2), 316–330.

Baron, David P and John A Ferejohn (1989). Bargaining in legislatures. *American Political Science Review 83*(04), 1181–1206.

Bernheim, B Douglas , Bezalel Peleg, and Michael D Whinston (1987). Coalition-proof nash equilibria i. concepts. *Journal of Economic Theory 42*(1), 1–12.

Cameron, Charles M and Lewis A Kornhauser (2008). Modeling collegial courts (3): Judicial objectives, opinion content, voting and adjudication equilibria.

Cameron, Charles M , Lewis A Kornhauser, and Giri Parameswaran (2019). Stare decisis and judicial log-rolls: a gains-from-trade model. *The RAND Journal of Economics 50*(3), 505–531.

Cardona, Daniel and Clara Ponsati (2011). Uniqueness of stationary equilibria in bargaining one-dimensional policies under (super) majority rules. *Games and Economic Behavior 73*(1), 65–75.

Carrubba, Cliff , Barry Friedman, Andrew D Martin, and Georg Vanberg (2012). Who controls the content of supreme court opinions? *American Journal of Political Science 56*(2), 400–412.

Chen, Ying and Hulya Eraslan (2018). Learning while setting precedents.

Cho, Seok-ju and John Duggan (2003). Uniqueness of stationary equilibria in a one-dimensional model of bargaining. *Journal of Economic Theory 113*(1), 118–130.

Cho, Seok-ju and John Duggan (2009). Bargaining foundations of the median voter theorem. *Journal of Economic Theory 144*(2), 851–868.

Clark, Tom , B Pablo Montagnes, and Jörg L Spenkuch (2018). Politics from the bench? ideology and strategic voting in the us supreme court.

Clark, Tom S and Benjamin Lauderdale (2010). Locating supreme court opinions in doctrine space. *American Journal of Political Science 54*(4), 871–890.

Compte, Olivier and Philippe Jehiel (2010). Bargaining and majority rules: A collective search perspective. *Journal of Political Economy 118*(2), 189–221.

Corley, Pamela C (2010). *Concurring opinion writing on the US Supreme Court*. SUNY Press.

Danelski, David J and Artemus Ward (2016). *The Chief Justice: Appointment and Influence*. University of Michigan Press.

Epstein, Lee and Jack Knight (1997). *The choices justices make*. Sage.

Fischman, Joshua B (2008). Decision-making under a norm of consensus: A structural analysis of three-judge panels.

Fischman, Joshua B (2011). Estimating preferences of circuit judges: A model of consensus voting. *The Journal of Law and Economics 54*(4), 781–809.

Hammond, Thomas H , Chris W Bonneau, and Reginald S Sheehan (2005). *Strategic behavior and policy choice on the US Supreme Court*. Stanford University Press.

Hettinger, Virginia A , Stefanie A Lindquist, and Wendy L Martinek (2003). Separate opinion writing on the united states courts of appeals. *American Politics Research 31*(3), 215–250.

Iaryczower, Matias and Matthew Shum (2012). The value of information in the court: Get it right, keep it tight. *The American Economic Review 102*(1), 202–237.

Jacobi, Tonja (2009). Competing models of judicial coalition formation and case outcome determination. *Journal of Legal Analysis 1*(2), 411–458.

Johnson, Timothy R , James F Spriggs, and Paul J Wahlbeck (2005). Passing and strategic voting on the us supreme court. *Law & Society Review 39*(2), 349–378.

Kalandrakis, Tasos (2010). Minimum winning coalitions and endogenous status quo. *International Journal of Game Theory 39*(4), 617–643.

Lax, Jeffrey R (2007). Constructing legal rules on appellate courts. *American Political Science Review 101*(3), 591–604.

Lax, Jeffrey R and Charles M Cameron (2007). Bargaining and opinion assignment on the us supreme court. *Journal of Law, Economics, and Organization 23*(2), 276–302.

Maltzman, Forrest and Paul J Wahlbeck (1996). May it please the chief? opinion assignments in the rehnquist court. *American Journal of Political Science*, 421–443.

O'Brien, David M (2016). *Judges on judging: Views from the bench*. Cq Press.

Parameswaran, Giri and Jacob Murray (2019). Limit equilibrium of unidimensional bargaining games: A core refinement.

Parameswaran, Giri and Hunter Rendleman (2019). Redistribution under general decision rules.

Posner, Richard A (2010). *How judges think*. Harvard University Press.

Rasmusen, Eric (1994). Judicial legitimacy as a repeated game. *Journal of Law, Economics, and Organization*, 63–83.

Romer, T. and H. Rosenthal (1978). Political resource allocation, controlled agendas, and the status quo. *Public choice 33*(4), 27–43.

Appendices

This supplementary Appendix is two parts. In part A, we present two extensions to the baseline model. The first considers a variant model in which bargaining failure results in a reversion to the legal *status quo*. The second consider a different variation, in which judges may change their dispositional votes after observing the proposed majority (and dissenting) opinions. As we make clear, the results from our baseline model continue to hold more-or-less under these variations. In part B, we present detailed proofs of the main results.

EXTENSIONS

Reversion to Status Quo

A legislature may propose changes to a given law repeatedly; however, unless one of those proposals is accepted, it is understood that the existing law continues to be in effect. The same cannot be said of courts. As we argued in Section 2, the mere fact that the court agrees to hear a case signals to the community that the legal landscape is apt to change, even if the court fails to implement that change in deciding the instant case. Thus, our preferred model specification does not include a *status quo* policy but instead requires that the court, through the bargaining process, eventually settle on a new policy.

Nevertheless, one might ask how our results would change if we instead assumed that failure to agree resulted in reversion to the *status quo ante*. The bargaining procedure would be amended as follows: in the event that a proposal is rejected, with probability δ a new proposer is selected and bargaining continues; however, with probability $1 - \delta$, the bargaining terminates (exogenously), and the policy reverts to the *status quo*.[20] This might represent the rare set of cases where no majority

[20] This is the bargaining protocol in Banks and Duggan (2006).

can be found to support any given opinion.

With this re-interpretation of the bargaining process, Proposition 1 (and all of the subsequent results) continue to hold true[21], replacing the disagreement utility with the utility of the *status quo* policy. Thus, our analysis is not only perfectly compatible with this alternative formulation, but demonstrates how to analyze it.

Of course, reversion to a *status quo* imposes different costs on different judges, depending on where the *status quo* stands in relation to their ideal policy. As such, the equilibrium policies will be different, even if the essential structure of the equilibrium is unchanged. One can show (see Banks and Duggan (2006)) that, if the *status quo* lies outside the core (i.e. $y_{sq} \notin [x^l, x^r]$), then with $\delta < 1$, there will be a range of equilibrium policies that are proposed in equilibrium, and that the social acceptance set becomes narrower as the likelihood that bargaining fails gets smaller (i.e. δ becomes larger). Moreover, as $\delta \to 1$, equilibrium proposals converge to a unique policy, characterized by the asymmetric Nash bargaining solution, as in Proposition 2. The analysis from section 3 carries through exactly as described.

However, if the *status quo* lies within the core (i.e. $y_{sq} \in [x^l, x^r]$), then for any δ, the only policy that is equilibrium consistent is the *status quo* itself. (It turns out that, in this case, the *status quo* policy exactly coincides with the asymmetric Nash bargaining solution, by construction, so Proposition 2 continues to hold, albeit trivially.)

Although we do not take up the issue of *certiorari* petitions in this paper, this last point may shed some light on the issue. Since whenever the *status quo* lies within the core, the court will fail to amend the existing rule, we should not expect the court to hear cases where such an outcome is

[21]A minor technical caveat: In the baseline framework, it was sufficient that the dispositional loss function l weakly satisfied the *IDID* property. Here, we strengthen that assumption, requiring the loss function to satisfy the *IDID* property strictly. Of particular interest, the loss function associated with the absolute value policy preferences that we highlighted in Example 1 only satisfies IDID weakly. However, the other cases presented all satisfy the strict condition.

likely to obtain. Moreover, since the core consists of the interval between the median judge's ideal, and the ideal policy of the other decisive judge (which, in the event of a unanimous dispositional vote, is also the median judge), it would be improvident for the court to grant *cert* on cases that where the *status quo ante* lies too close to the median judge's ideal policy. Furthermore, to the extent that the Court does agree to here a case, we should expect larger coalitions and more strategic voting, since the size of the core (which determines the likelihood of failing to amend the existing rule) is decreasing in the size of the majority coalition.

Even when the *status quo* policy lies outside of the core (so that policies are chosen through a genuine process of bargaining), its location affects the policies that will be chosen in equilibrium. Interestingly, as the status quo policy becomes more extreme, the policy that is implemented is likely to be more moderate (in the sense of being closer to the 'middle' of the core), *ceteris paribu*s (see Parameswaran and Rendleman (2019)). Thus, policy-making by the court exhibits path dependence, with existing rules shaping the sorts of rules that courts can implement in the future.

Dissents and Competition for the Dispositional Majority

In the baseline model, we assumed that, once chosen, the composition of the dispositional majority remained fixed. Since there is little evidence that dispositional coalitions shift between the initial conference and the Court's rendering of it final decision, we hold this assumption to be reasonable, as an empirical matter. Nevertheless, it may be objected that this result ought to be a consequence of our model, rather than an assumption. In this sub-section, we consider a variant model in which stable dispositional coalitions arise in equilibrium.

Before outlining the variant model and results, let us briefly acknowledge the implications of our baseline approach. In the baseline model, since the dispositional majority was fixed in the first stage, the consequence of proposing a relatively 'extreme' policy in the second stage was simply that the policy would be rejected and a counter-proposal made. However, if dispositional coalitions were allowed to change, there may be an additional consequence; an extreme proposal might cause sufficiently many judges to switch their dispositional votes, such that the original majority is lost. The threat of such defection creates an additional incentive for judges to moderate their proposals. It is this additional incentive that we seek to explore.

We modify the model as follows: after the initial dispositional vote, the judges divide into majority and minority dispositional coalitions. As before, the most senior judge in the majority assigns to some judge in the majority, the task of writing a majority opinion, and this opinion may be refined through a sequence of counter-proposals. Similarly, the most senior judge in the minority assigns to some judge in the minority, the task of writing a dissent. Having observed the two opinions, the judges then take a second dispositional vote, with the understanding that whichever opinion receives a majority will automatically become the opinion of the court.[22] We retain the baseline assumption that policy-making is purely consequentialist —opinion location matters only insofar as it affects the judges' actual policy utility. Thus, the role of the dissent is not as an expression of the minority's ideal rule, but as a competing potential majority opinion. The location of the dissent affects utility only if it succeeds in causing the disposition of the court to switch.[23]

For concreteness, suppose $x_{med} < z$, so that the median judge's ideal disposition is $d = 1$. Consider the $d = 0$ and $d = 1$ dispositional coalitions. The former must agree on an opinion $y_0 \geq z$ and the latter must write an opinion $y_1 \leq z$.

We briefly note some features of incentives in this new setting. First, every judge who voted sincerely would rather moderate their side's opinion to guarantee that they were in the eventual majority, than

[22]In principle, if the dispositional coalitions change, we could allow for new majority and dissenting opinions to be drafted, and for this process to continue *ad infinitum*, until a pair of opinions arise for which the dispositional coalitions are stable. It suffices, however, in equilibrium, that there be a single additional round of dispositional voting.

[23]As we noted in footnote 16, in a dynamic model, there might be a role for a dissent that has no immediate policy consequence, but which sets the basis for a different policy to be adopted if the court revisits the issue in the future.

write an opinion that results in the eventual majority going to the other side. This should be intuitive; the most moderate opinion consistent with one's ideal disposition is preferred to any opinion that rationalizes the opposite disposition. Thus, in the competition over opinions, there is a strong force that pushes each coalition to moderate its opinion in order to win (or retain) a majority.

Second, since the support of the median judge is sufficient to win a majority, both sides will 'moderate' their opinions with a view to earning the support of the median judge. Notice that the $d = 1$ coalition has a distinct advantage in this regard. They can always offer the median judge her ideal policy $y_1 = x_{med}$, whereas dispositional consistency restricts the $d = 0$ coalition to at best offer $y_0 = z > x_{med}$. Thus, in equilibrium, the $d = 1$ coalition will always prevail —the disposition of the court will coincide with the ideal disposition of the median judge.

Third, although the majority opinion must be close to the median judge's ideal, it need not coincide with the median's ideal policy. A majority opinion is *incentive compatible* if it is weakly preferred by the median judge to the dissenting opinion. In equilibrium, the median judge must do at least as well by joining the $d = 1$ coalition, as if she joined a $d = 0$ coalition offering the most moderate policy satisfying dispositional consistency (i.e. when the dispositional consistency requirement is binding on the dissent). Let $\zeta(z)$ be the policy (with $\zeta(z) < x_{med} < z$) having the property that $u_P(\zeta(z), x_{med}) = u_P(z, x_{med}) + \alpha u_D(z, x_{med})$. The median judge would be indifferent between voting sincerely and endorsing opinion $\zeta(z)$, and voting strategically and endorsing opinion z (the most moderate policy that the rationalizes the opposite disposition). Any policy in the interval $[\zeta(z), z]$ is thus equilibrium incentive compatible for the $d = 1$ coalition.

Recall, $d(z)$ denotes the disposition of the court if all judges voted sincerely, and $M(z)$ denotes the majority coalition when there is sincere voting. By construction, the sincere disposition must coincide with the ideal disposition of the median judge. The above points, taken together, imply the following:

Proposition 4. *The game with competing opinions admits a unique CCPAE* (d^*, M^*) *satisfying:*

1. *The equilibrium disposition coincides with the sincere disposition, i.e.* $d^* = d(z)$.
2. *All judges who sincerely agree with the median will vote sincerely, while some judges who sincerely disagree may vote strategically, i.e.* $M(z) \subseteq M^*$.
3. *The policies proposed in the policy-making stage are given by a modified version of Proposition 1, in which proposals must additionally satisfy the incentive compatibility condition. (Formally, an equilibrium proposal* y *must satisfy:* $u_P(y, x_{med}) \geq u_P(z, x_{med}) + \alpha u_D(z, x_{med})$*.)*
4. *The equilibrium is sustained by a (threatened) dissent,* $y_{diss} = z$.

A few comments are worth noting. First, we stress that most of the results from the baseline model continue to hold, even after adding competing dissents and allowing the composition of dispositional coalitions to change. The policy-making results (section 3) are qualitative unchanged, and require only a minor modification in the addition of the incentive compatibility constraint. Proposition 1, appropriately modified, will continue to imply that whenever $\delta < 1$, there will be range of potential majority opinions, reflecting a degree of agenda control by the opinion authors. Furthermore, per Proposition 2, as $\delta \to 1$, these opinions all converge to a unique policy that generically *does not* coincide with the median judge's ideal, and which is characterized by the (incentive compatibility constrained) Nash bargaining solution. Most of the results from section 4 (dispositional voting) also carry over, including Proposition 3 and Lemmas 2 and 3. The median judge remains dispositionally pivotal (although with competing dissents, she is guaranteed to vote sincerely), and judges whose ideal disposition coincides with the median judge's will always vote sincerely. Moreover, judges who sincerely disagree may vote strategically to participate in policy-making, and the likelihood of strategic voting decreases as expressive utility becomes more salient.

Equilibrium in the model with competing opinions differs from the baseline in two ways. First, at the adjudication stage, there is now a unique CCPAE, which renders the results in Corollary 1 moot. Moreover, the disposition in this unique CCPAE coincides with median judge's ideal. Second, at the policy-making stage, there is an additional constraint (incentive compatibility) that affects the set of profile of policies that may be offered in equilibrium. Indeed, adding this constraint is sufficient to cause the results of the baseline and variant models to coincide.

Second, we briefly note that the equilibrium does not require that a dissent actually be constructed as described —simply that the minority can credibly threaten to write such a policy (which they can).

Finally, the equilibrium with competing dissents may be thought of as a 'median voter theorem with frictions'. There is clearly a (Bertrand-competition-like) force that pushes the equilibrium policy closer to the median judge's ideal. However, the requirement that the dissent be dispositionally consistent, along with the expressive cost of voting insincerely, make the dissenting opinion an imperfect substitute to the majority opinion, from the perspective of the median judge. This allows other judges in the dispositional majority to pull the majority opinion slightly away from the median's ideal, subject to incentive compatibility. Hence, a range of majority opinions are equilibrium consistent and policy needn't converge all the way to the median's ideal.

PROOFS

Proof of Proposition 1. The proof is similar to that in Parameswaran and Murray (2019). Since u_P is non-concave, we must first establish that equilibria must be in no-delay pure strategies. Let

$$\begin{aligned} v_P(F(y);F(x)) &= u_P\left(F^{-1}(F(y));F^{-1}(F(x))\right) = u_P(y,x) \\ &= -\left|\int_{F^{-1}(F(x))}^{F^{-1}(F(y))} l(z-x)dF(z)\right| \end{aligned}$$

be the policy utility after re-scaling the policy space. Notice that v_P is concave in $F(y)$:

$$\frac{\partial^2 v_P}{\partial F(y)^2} = -\left|l'\,(y-x))\cdot\frac{1}{f(F(y))}\right| < 0$$

Now, take any (possibly mixed) profile of strategies in the continuation game. Let $\sigma(y,t)$ be the implied distribution over outcomes, where $\sigma(y,t)$ is the probability that policy y is agreed to at time t. Let $\Delta u_P(y, x) = u_P(y, x) - u_P(D, x)$ be the utility gain over disagreement of policy y for a judge with ideal policy x. Similarly, define $\Delta v_P(F(y), F(x))$. Let $\hat{y}$ be the policy defined by: $F(\hat{y}) = \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y), t) \cdot \delta^t F(y) dy$.

Then, the judge i's continuation payoff (over disagreement) if the current proposal is rejected is:

$$\begin{aligned}
\delta \Delta U(x^i) &= \delta \sum_{t=0}^{\infty} \int_{\underline{x}}^{\overline{x}} \sigma(y, t) \cdot \delta^t \Delta u_P(y, x^i) dy \\
&= \delta \left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y), t) \cdot \delta^t dy \right) \cdot \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \frac{\sigma(F(y), t) \cdot \delta^t}{\left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y), t) \cdot \delta^t dy \right)} \Delta v_P(F(y), F(x^i)) dy \\
&\leq \delta \left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y), t) \cdot \delta^t dy \right) \cdot \Delta v_P(F(\hat{y}), F(x^i)) \\
&< \Delta u_P(\hat{y}, x^i)
\end{aligned}$$

where we use the facts that v_P is concave, and that $\delta \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y), t) \cdot \delta^t dy \leq \delta < 1$. Hence, there is a policy $\hat{y}$ that is strictly preferred by every judge to the continuation game. It is immediate, then, that there is a proposal for every judge that is socially acceptable and preferable to the continuation game. Moreover, since u_P is strictly quasi-concave, this policy is unique. Hence, every equilibrium must be in pure strategies and no-delay.

The acceptance set for any judge i is $A_i = \{y \in [\underline{x}, \overline{x}] \,|\, \Delta u_P(y, x^i) \geq \delta \Delta U(x^i)\}$. Since $u_P(y; x^i)$ is strictly quasi-concave in y, each individual acceptance set is an interval $A_i = [\underline{y}_i, \overline{y}_i]$. Let $C \subset \{1, \ldots, m\}$ be any coalition containing at least k members. Then, the coalitional acceptance set $A_C = \cap_{i \in C} A_i$ is also an interval. Moreover, since each A_i (and thus each A_C) contains $\hat{y}$, the social acceptance set $A = \cup_C A_C$ must be an interval as well. Denote $A = \left[\underline{y}, \overline{y}\right]$.

Given this social acceptance set, the optimal offers for each agent are:

$$y_i = \begin{cases} \underline{y} & x^i \leq \underline{y} \\ x^i & x^i \in \left(\underline{y}, \overline{y}\right) \\ \overline{y} & x^i \geq \overline{y} \end{cases}$$

For notational convenience, we often denote $u_P(y, x^i)$ by $u_i(y)$. For any $x \in X$, let $P(x) = \sum_{x_i \leq x} p_i$. (The proof allows for p_i's to be different, although we typically focus on the case of $p_i = \frac{1}{m}$.) Then, given social acceptance set $\left[\underline{y}, \overline{y}\right]$, the expected utility of each judge i is:

$$U_i\left(\underline{y}, \overline{y}\right) = P\left(\underline{y}\right) u_i\left(\underline{y}\right) + \sum_{j:\, x^j \in \left(\underline{y}, \overline{y}\right)} p_j u_i\left(x^j\right) + \left(1 - P\left(\overline{y}\right)\right) u_i\left(\overline{y}\right)$$

The remainder of the proof proceeds in two steps. First, we show that in any equilibrium, $\underline{y} = \underline{y}_r$ and $\overline{y} = \overline{y}_l$. Next, using this fact, we show that the equilibrium is a fixed point of a mapping, and that the mapping admits a unique fixed point. This suffices to prove uniqueness of the equilibrium.

Step 1. For any player i, suppose $u_i\left(\underline{y}\right) \leq (1-\delta)\, u_i(D) + \delta U_i\left(\underline{y}, \overline{y}\right)$ — i.e. that $\Delta u_i(\underline{y}) < \delta \Delta U_i(\underline{y}, \overline{y})$. Since policy preferences satisfy the single crossing property, it must be that: $\Delta u_j\left(\underline{y}\right) < \delta \Delta U_j\left(\underline{y}, \overline{y}\right)$ for any j with $x^j > x^i$. To see this, suppose not; i.e. suppose $\Delta u_j\left(\underline{y}\right) \geq \delta U_j\left(\underline{y}, \overline{y}\right)$. Then:

$$\Delta u_i(\underline{y}) - \Delta u_j(\underline{y}) < \delta[\Delta U_i\left(\underline{y}, \overline{y}\right) - \Delta U_j\left(\underline{y}, \overline{y}\right)]$$

Recall, by the single crossing condition, that $x^i < x^j$ implies $\frac{\partial}{\partial y}(\Delta u_i - \Delta u_j) \leq 0$ (see footnote 8).

Then:

$$\Delta U_i\left(\underline{y},\overline{y}\right)-\Delta U_j\left(\underline{y},\overline{y}\right)=P\left(\underline{y}\right)\left[\Delta u_i\left(\underline{y}\right)-\Delta u_j\left(\underline{y}\right)\right]+\sum_{j:\,x^j\in\left(\underline{y},\overline{y}\right)}p_j\left[\Delta u_i\left(x^j\right)-\Delta u_j\left(x^j\right)\right]$$

$$+\left(1-P\left(\overline{y}\right)\right)\left[\Delta u_i\left(\overline{y}\right)-\Delta u_j\left(\overline{y}\right)\right]$$

$$\leq \Delta u_i\left(\underline{y}\right)-\Delta u_j\left(\underline{y}\right)$$

But by assumption, $\Delta u_i\left(\underline{y}\right)-\Delta u_j\left(\underline{y}\right)<\delta\left[\Delta U_{i}-_{j}\right]\leq\delta\left(\Delta u_i(\underline{y})-\Delta u_j(\underline{y})\right)$, which is a contradiction. Hence, $\Delta u_i\left(\underline{y}\right)\leq\delta\Delta U_i\left(\underline{y},\overline{y}\right)$ implies that $\Delta u_j\left(\underline{y}\right)<\delta\Delta U_j\left(\underline{y},\overline{y}\right)$ whenever $x^j>x^i$. We can similarly show that $\Delta u_i\left(\overline{y}\right)\leq\delta\Delta U_i\left(\underline{y},\overline{y}\right)$ implies $\Delta u_j\left(\overline{y}\right)<\delta\Delta U_j\left(\underline{y},\overline{y}\right)$ whenever $x^j<x^i$.

Suppose $\underline{y}<\underline{y}_r$, then any proposal $y\in\left[\underline{y},\underline{y}_r\right)$ will be rejected by agent r and all agents $j>r$. But since $r=k$, this implies that fewer than k agents will accept the proposal, which means it cannot be in the acceptance set. Hence $\underline{y}\geq\underline{y}_r$. Suppose $\underline{y}>\underline{y}_r$. Take any proposal $y\in\left(\underline{y}_r,\underline{y}\right)$. By construction $\Delta u_r\left(y\right)>\delta\Delta U\left[\underline{y},\overline{y}\right]$, and so $u_j\left(y\right)>\delta\Delta U\left[\underline{y},\overline{y}\right]$ for all agents $j<r$. But since $r=k$, this implies that at least k agents will accept proposal y. But this contradicts the assumption that y is outside the acceptance set. Hence $\underline{y}=\underline{y}_r$. We can similarly show that $\overline{y}=\overline{y}_l$.

Step 2. We now show that the equilibrium exists and is unique. For each i, define $\underline{\zeta}_i\left(z\right)=\min_{y\in X}\left\{y\leq x_i|\,\Delta u_i\left(y\right)\geq\delta\Delta U_i\left(y,z\right)\right\}$ and $\overline{\zeta}_i\left(z\right)=\max_{y\in X}\left\{y\geq x_i|\,\Delta u_i\left(y\right)\geq\delta\Delta U_i\left(z,y\right)\right\}$. Since u_i is continuous and X compact, then $\underline{\zeta}_i$ and $\overline{\zeta}_i$ are both continuous. Note also that:

$$\underline{\zeta}_j'\left(y\right)=\begin{cases}\frac{\delta(1-P(y))}{1-\delta P\left(\underline{\zeta}_j(y)\right)}\cdot\frac{u_j'(y)}{u_j'\left(\underline{\zeta}_j(y)\right)} & \underline{\zeta}_j\left(y\right)>\underline{x}\\ 0 & \underline{\zeta}_j\left(y\right)=\underline{x}\end{cases}$$

and:

$$\overline{\zeta}_i'(y) = \begin{cases} \frac{\delta P(y)}{1-\delta+\delta P\left(\overline{\zeta}_i(y)\right)} \cdot \frac{u_i'(y)}{u_i'\left(\overline{\zeta}_i(y)\right)} & \overline{\zeta}_i(y) < \overline{x} \\ 0 & \overline{\zeta}_i(y) = \overline{x} \end{cases}$$

By the previous step, we know that $\overline{y} = \overline{y}_l$ and $\underline{y} = \underline{y}_r$. Hence, $\overline{y} = \overline{\zeta}_l\left(\underline{y}\right)$ and $\underline{y} = \underline{\zeta}_r(\overline{y})$. Let $H(y) = \overline{\zeta}_l\left(\underline{\zeta}_r(y)\right)$. H is continuous since $\underline{\zeta}_r$ and $\overline{\zeta}_l$ are both continuous. It follows that if $\left[\underline{y}, \overline{y}\right]$ is an equilibrium acceptance set, then $\overline{y}$ is a fixed point of H, and $\underline{y} = \underline{\zeta}_r(\overline{y})$. Since X is compact and H is continuous and onto X, it follows by Brouwer's fixed point theorem that H admits a fixed point $\overline{y}$. Hence, an equilibrium of the bargaining exists.

To establish that H has a unique fixed point, it suffices to show that $H'(\overline{y}) < 1$ for any $\overline{y}$ that is a fixed point. (If there exist multiple fixed points, then $H' \geq 1$ for at least one fixed point.) By construction:

$$H'(\overline{y}) = \begin{cases} A(\overline{y}) \cdot \frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})} \cdot \frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)} & \underline{x} < \underline{y} \leq \overline{y} < \overline{x} \\ 0 & \underline{y} = \underline{x} \text{ or } \overline{y} = \overline{x} \end{cases}$$

where $\underline{y} = \underline{\zeta}_r(\overline{y}) < \min\{x_r, \overline{y}\}$, and $A(y) = \frac{\delta P\left(\underline{\zeta}_r(\overline{y})\right)}{1-\delta+\delta P(\overline{y})} \frac{\delta(1-P(\overline{y}))}{1-\delta P\left(\underline{\zeta}_r(\overline{y})\right)} \in (0,1)$.

Suppose $H(\overline{y}) \geq 1$. Then at least one of $\left|\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})}\right| > 1$ or $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| > 1$. There are several cases to consider. First, suppose $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| > 1$. Since $\underline{y} < \min\{x_r, \overline{y}\}$ then $u_r'\left(\underline{y}\right) > 0$. If $\underline{y} \leq \overline{y} \leq x_r$, then $0 \leq u_r'(\overline{y}) \leq u_r'\left(\underline{y}\right)$, which contradicts $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| > 1$. Hence $\underline{y} < x_r < \overline{y}$, and so $u_r'(\overline{y}) < 0$. Suppose additionally $x_l \leq \underline{y} < \overline{y}$. Then $u_l'\left(\underline{y}\right) < 0$ and $u_l'(\overline{y}) < 0$. Hence $\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)} < -1$, and $\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})} > 0$, and so $H < 0$, which cannot be. Hence $\underline{y} < x_l \leq x_r < \overline{y}$, and so:

$$\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})} \cdot \frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)} = \frac{-l\left(\underline{y} - x_l\right)}{l(\overline{y} - x_l)} \cdot \frac{l(\overline{y} - x_r)}{-l\left(\underline{y} - x_r\right)} \leq 1$$

since $l\left(z\right)$ is weakly increasing for $z < 0$ and weakly decreasing for $z > 0$. Hence $H < 1$, which cannot be, and so $\left|\frac{u_r'\left(\overline{y}\right)}{u_r'\left(\underline{y}\right)}\right| \leq 1$.

Next, suppose that $\left|\frac{u_l'\left(\underline{y}\right)}{u_l'\left(\overline{y}\right)}\right| > 1$.Since $\overline{y} > \max\left\{x_l, \underline{y}\right\}$, then $u_l'\left(\overline{y}\right) < 0$. If $x_l \leq \underline{y} \leq \overline{y}$, then $u_l'\left(\overline{y}\right) \leq u_l\left(\underline{y}\right) \leq 0$, which contradicts that $\left|\frac{u_l'\left(\underline{y}\right)}{u_l'\left(\overline{y}\right)}\right| > 1$. Hence $\underline{y} < x_l < \overline{y}$, and so $u_l'\left(\underline{y}\right) > 0$. Suppose additionally that $\underline{y} < \overline{y} \leq x_r$. Then $u_r'\left(\underline{y}\right) > 0$ and $u_r'\left(\overline{y}\right) > 0$. Hence $\frac{u_r'\left(\overline{y}\right)}{u_r'\left(\underline{y}\right)} > 0$, and $\frac{u_l'\left(\underline{y}\right)}{u_l'\left(\overline{y}\right)} < -1$, and so $H < 0$, which cannot be. Hence $\underline{y} < x_l \leq x_r < \overline{y}$. But we know that this implies $H < 1$, which also cannot be. Hence our initial supposition was wrong; $H'\left(\overline{y}\right) \ngeq 1$. Hence, $H' < 1$ and so H admits a unique fixed point. □

Proof of Lemma 1 . Recall, the acceptance set is $A = [\underline{y_r}, \overline{y_l}]$, where $\underline{y_r} = \min\{y \geq \underline{x} \mid \Delta u_r(y) \geq \delta \Delta U_r(y, \overline{y_l})\}$, and $\overline{y_l} = \max\{y \leq \overline{x} \mid \Delta u_l(y) \geq \delta \Delta U_l(\underline{y_r}, y)\}$. Now, by construction $\Delta u_l(\underline{y_r}) \geq \Delta u_l(\overline{y_l})$, since l will accept $\underline{y_r}$. Then, since u is strictly quasi-concave, $\Delta u_l(y) > \Delta u_l(\overline{y_l})$ for all $y \in (\underline{y_r}, \overline{y_l})$. Similarly, $\Delta u_r(y) > \Delta u_r(\underline{y_r})$ for all $y \in (\underline{y_r}, \overline{y_l})$. Hence $\Delta U_l(\underline{y_r}, \overline{y_l}) > \Delta u_l(\overline{y_l})$ and $\Delta U_r(\underline{y_r}, \overline{y_l}) > \Delta u_r(\underline{y_r})$ whenever $\underline{y_r} < \overline{y_l}$.

Now, for every $\delta < 1$, $\frac{\Delta u_l(\overline{y_l})}{\Delta U_l(\underline{y_r}, \overline{y_l})} = \delta = \frac{\Delta u_r(\underline{y_l})}{\Delta U_r(\underline{y_r}, \overline{y_l})}$, and so as $\delta \to 1$, we need $\Delta U_l(\underline{y_r}, \overline{y_l}) - \Delta u_l(\overline{y_l}) \to 0$ and $\Delta U_r(\underline{y_r}, \overline{y_l}) - \Delta u_r(\underline{y_r}) \to 0$. But this requires $\overline{y_l} - \underline{y_r} \to 0$. Hence $A = [\underline{y_r}, \overline{y_l}] \to [\mu, \mu]$ as $\delta \to 1$. □

Proof of Proposition 2 . Take any $i \in \{1, ..., m\}$, and suppose $\mu \in \left(x^{i-1}, x^i\right)$. Then, by Lemma 1, there exists $\bar{\delta} < 1$ s.t. for $\delta > \bar{\delta}$, $x^{i-1} < \underline{y_r}\left(\delta\right) < \overline{y_l}\left(\delta\right) < x^i$. (For clarity, we make explicit the dependence of $\underline{y_r}$ and $\overline{y_l}$ on δ.) Then, by Proposition 1, all judges $j \in \{1, .., i-1\}$ will propose $\underline{y_r}$ and all judges $j \in \{i, .., n\}$ will propose $\overline{y_l}$. Again by Proposition 1, this implies that:

$$\Delta u_r\left(\underline{y_r}\right) = \delta\left[\left(1 - P_i\right)\Delta u_r\left(\underline{y_r}\right) + P_i \Delta u_r\left(\overline{y_l}\right)\right] \tag{1}$$

$$\Delta u_l\left(\overline{y_l}\right) = \delta\left[\left(1 - P_i\right)\Delta u_l\left(\underline{y_r}\right) + P_i \Delta u_l\left(\overline{y_l}\right)\right] \tag{2}$$

where $P_i = \sum_{j \geq i} p_j$. By the implicit function theorem, this system of equations pins down $\underline{y_r}$ and $\overline{y_l}$ in terms of the model parameters.

Now, let $E[y] = (1 - P_i)\underline{y_r} + P_i \overline{y_l}$. Note, by construction, that $\underline{y_r} < E[y] < \overline{y_l}$. Then $\overline{y_l} - E[y] = \frac{1-P_i}{P_i}\left(E[y] - \underline{y_r}\right)$. We affect the following change of variables: Let $\varepsilon = E[y] - \underline{y_r}$. Then, we have: $\underline{y_r} = E[y] - \varepsilon$ and $\overline{y_l} = E[y] + \frac{1-P_i}{P_i}\varepsilon$. Equations (1) and (2) become:

$$(1 - \delta(1 - P_i))\,\Delta u_r(E[y] - \varepsilon) = \delta P_i \Delta u_r\left(E[y] + \frac{1 - P_i}{P_i}\varepsilon\right) \tag{3}$$

$$(1 - \delta P_i)\,\Delta u_l\left(E[y] + \frac{1 - P_i}{P_i}\varepsilon\right) = \delta(1 - P_i)\,\Delta u_l(E[y] - \varepsilon) \tag{4}$$

By the implicit function theorem, and since u is continuously differentiable, we have:

$$\begin{bmatrix} (1 - \delta(1 - P_i))\,u_r'\left(\underline{y_r}\right) - \delta P_i u_r'(\overline{y_l}) & -(1 - \delta(1 - P_i))\,u_r'\left(\underline{y_r}\right) - \delta(1 - P_i)\,u_r'(\overline{y_l}) \\ (1 - \delta P_i)\,u_l'(\overline{y_l}) - \delta(1 - P_i)\,u_l'\left(\underline{y_r}\right) & \left(\frac{1-P_i}{P_i} - \delta(1 - P_i)\right)u_l'(\overline{y_l}) + \delta(1 - P_i)\,u_l'\left(\underline{y_r}\right) \end{bmatrix} \begin{pmatrix} \frac{\partial E[y]}{\partial \delta} \\ \frac{\partial \varepsilon}{\partial \delta} \end{pmatrix} =$$

$$\begin{pmatrix} (1 - P_i)\,\Delta u_r\left(\underline{y_r}\right) + P_i \Delta u_r(\overline{y_l}) \\ P_i \Delta u_l(\overline{y_l}) + (1 - P_i)\,\Delta u_l\left(\underline{y_r}\right) \end{pmatrix}$$

Taking limits as $\delta \longrightarrow 1$, we have:

$$\begin{bmatrix} 0 & -u_r'(\mu) \\ 0 & \frac{1-P_i}{P_i}u_l'(\mu) \end{bmatrix} \begin{pmatrix} \lim_{\delta \to 1} \frac{\partial E[y]}{\partial \delta} \\ \lim_{\delta \to 1} \frac{\partial \varepsilon}{\partial \delta} \end{pmatrix} = \begin{pmatrix} u_r(\mu) \\ u_l(\mu) \end{pmatrix}$$

These imply that:

$$\lim_{\delta \to 1} \frac{\partial \varepsilon}{\partial \delta} = -\frac{u_r(\mu)}{u_r'(\mu)} = \frac{P_i}{1 - P_i}\frac{u_l(\mu)}{u_l'(\mu)}$$

The second equality provides an equation that uniquely defines the limit equilibrium.

Next, we note that equation defining μ_i coincides with the first order condition of the i^{th} Nash Bargaining problem. Recall, that problem was: $\max_{y \in X} (\Delta u_l(y))^{1-P_i} (\Delta u_r(y))^{P_i}$. Since utilities are concave (after rescaling the space), the maximizer must be the solution to the first order condition:

$(1-P_i)\frac{u^l_{P,y}\left(b_{i-1,i}\right)}{u^l_P\left(b_{i-1,i}\right)}+P_i\frac{u^r_{P,y}\left(b_{i-1,i}\right)}{u^r_P\left(b_{i-1,i}\right)}=0$. Re-arranging gives the desired result.

Notice that $b_{i-1,i}$ is increasing in P_i. (To see this, re-arrange the first order condition to give: $\frac{u'_l\left(b_{i-1,i}\right)}{u'_r\left(b_{i-1,i}\right)}\cdot\frac{u_r\left(b_{i-1,i}\right)}{u_l\left(b_{i-1,i}\right)}=-\frac{P_i}{1-P_i}$. We know that $b\in\left[x^l,x^r\right]$. By single-peakedness, over this region we know that $u_l(b)$ is strictly decreasing in b and $u_r(b)$ is strictly increasing in b, and so $\frac{u_r(b)}{u_l(b)}$ is strictly decreasing in b. Similarly, by concavity (after transformation) of u, $u'_l(b)$ is decreasing in b and $u'_r(b)$ is increasing in b, and so $\frac{u'_r(b)}{u'_l(b)}$ is weakly decreasing in b. Hence, the left hand term is strictly decreasing in b. The right hand term is also strictly decreasing in P. Hence, as P increases, so must b.) Then, since P_i is decreasing in i, it follows that $b_{i-1,i}$ is decreasing is as well.

Since we conjectured $\mu\in\left(x^{i-1},x^i\right)$, then the limit equilibrium policy coincides with i^{th} Nash Bargaining solution provided that $x^{i-1}<b_{i-1,i}<x^i$. Now, since x^i is increasing and $b_{i-1,i}$ is decreasing in i, then by the definition of i^*, $x^i<b_{i,i+1}$ for all $i<i^*$ and $x^i\geq b_{i,i+1}$ for all $i\geq i^*$. Moreover, for $i<i^*$, $x^{i-1}\leq x^i<b_{i,i+1}\leq b_{i-1,i}$, which is inconsistent. Similarly, for $i>i^*$, $b_{i-1,i}\leq x^{i-1}\leq x^i$, which is inconsistent. Hence, if $b_{i-1,i}\in\left(x^{i-1},x^i\right)$, then $i=i^*$. Note however, that the converse need not be true. Setting $i=i^*$ gives two possibilities: (i) $x^{i^*-1}<b_{i^*-1,i^*}<x^{i^*}$, or (ii) $x^{i^*-1}\leq x^{i^*}\leq b_{i^*-1,i^*}$ (with at least one inequality strict). The former case is equilibrium consistent, and since the equilibrium is unique, we have $\mu=b_{i^*-1,i^*}$.

Suppose the latter case prevails. It follows that the limit equilibrium is not contained in any of the open intervals $\left\{(x^{i-1},x^i)\right\}_{i=1}^{m-1}$, and so $\mu\in\left\{x^1,...,x^m\right\}$. (In fact, since $\underline{y_r}<x^r$ and $\overline{y_l}>x^l$ for all δ, and since $\lim_{\delta\to1}\underline{y_r}=\mu=\lim_{\delta\to1}\overline{y_l}$, then $x^l\leq\mu\leq x^r$, and so $\mu\in\left\{x^l,...,x^r\right\}$.) Suppose $\mu=x^i$ for some $i\in\{l,...,r\}$. Let $I=\{j|x^j=x^i\}$ and denote $I=\{i^-,...,i^+\}$, where $i^-\leq j\leq i^+$ for all $j\in I$. (Obviously, I may be a singleton, in which case $i^-=i=i^+$.) Let $\Pi_i^-=\sum_{j<i^-}p_j$ and $\Pi_i^+=\sum_{j>i^+}p_j$ and $\Pi_i=\sum_{j\in I}p_j$. Then, for δ sufficiently large, (1) becomes:

$$u_r(\underline{y}_r)=\delta\left[\Pi_i^-u_r(\underline{y}_r)+\Pi_iu_r(x^i)+\Pi_i^+u_r(\overline{y}^l)\right]$$

Since $\underline{y}_r < x^i < \overline{y}^l$, there exists $\tau \in (0,1)$ s.t. $x^i = \tau \underline{y}^r + (1-\tau)\overline{y}^l$. We can write (1) as:

$$\begin{aligned} u_r(\underline{y}^r) = \delta & \left[(\Pi_i^- + \Pi_i \tau) u_r(\underline{y}^r) + (\Pi_i^+ + \Pi_i(1-\tau)) u_r(\overline{y}_l)\right] \\ & + \delta \left[\Pi_i \tau (u_r(\underline{y}_r) - u_r(x^i)) + \Pi_i (1-\tau)(u_r(\overline{y}^l) - u_r(x^i))\right] \end{aligned} \tag{5}$$

Notice (5) is the sum of two terms, with the first term being analogous to the expression in (1), and the second term being a 'correction' term.

We repeat the procedure for equation (2), and then apply the change of basis method above, and take limits as $\delta \to 1$. Since $\underline{y}^r \to x^i$ and $\overline{y}^l \to x^i$, the 'correction' term in (5) goes to zero. It follows that $\mu = b(\rho^*)$, where $\rho^* = \Pi_i^+ + \Pi_i(1 - \lim_{\delta \to 1} \tau(\delta))$. Now, there must be some k s.t. $b_{k,k+1} < b(\rho^*) = x^i < b_{k-1,k}$. Moreover, it must be that $k \in I$, since $b_{i^+,i^++1} < b(\rho^*) < b_{i^--2,i^--1}$, by construction. But then, we can choose i appropriately s.t. $b_{i,i+1} < x^i < b_{i-1,i}$. But this requires $i = i^*$. □

Proof of Lemma 2. Let z be an arbitrary case. Suppose $d^* = 0$. (The other scenario is analogous.) Recall $M^0 = \{j | x^j > z\}$. Moreover, all feasible second stage policies must satisfy $y \geq z$. Suppose there is a j, such that $j \in M^0$ and $j \notin M^*$. Then the payoff to j of choosing $d = 1$ must exceed that of choosing $d = 1$, which implies:

$$\left[u_P(\gamma(M), x^j) - u_P\left(\gamma(M \cup \{j\}), x^j\right)\right] + \alpha l(z - x^j) > 0$$

By assumption 1, the term in square brackets is non-positive, since joining the coalition cannot make the policy worse from j's perspective. Moreover, the second term is negative by construction. Hence the LHS is negative, which is a contradiction. Hence $j \in M^*$. □

Lemma 4. *Let (d, M) and (d, M') both be adjudication (Nash) equilibria, and suppose $M \subset M'$. Then (d, M) is not coalition-proof.*

Proof of Lemma 4. Suppose (d, M) and (d, M') are both adjudication (Nash) equilibria, with

$M \subset M'$. Since M and M' are both equilibrium coalitions, it (generically) must be that $|M'| \geq |M+2|$, where $|X|$ denotes the cardinality of set X. (To see this, note that if $M' = M \cup \{i\}$ where $i \notin M$, then it must be that judge i is exactly indifferent between joining the majority coalition or not; otherwise, i would have a strictly improving unilateral deviation. This indifference is non-generic and requires an exact alignment of the case, the equilibrium policies chosen by the respective coalitions, and the salience parameter α.)

Note by Lemma 2 that $M^d(z) \subseteq M \subset M'$. WLOG, suppose $d = 1$. Then, by part 1 of Assumption 1, $\gamma(M) \leq \gamma(M' \setminus \{j\}) \leq \gamma(M')$ for every $j \in M' \setminus M$, since $M \subset M' \setminus \{j\}$. Moreover, for all $j \in M' \setminus M$, $\gamma(M) \leq \gamma(M') \leq z < x^j$. Now, since M' is a Nash equilibrium coalition, then $u_P(\gamma(M'), x^j) + \alpha l(z, x^j) \geq u_P(\gamma(M' \setminus \{j\}), x^j)$ for each $j \in M' \setminus M$, and given the above ordering, we know that $u_P(\gamma(M' \setminus \{j\}), x^j) \geq u_P(\gamma(M), x^j)$. Hence $u_P(\gamma(M'), x^j) + \alpha l(z, x^j) \geq u_P(\gamma(M), x^j)$ for all $j \in M' \setminus M$, and this inequality will generically be strict for some j. Hence, the joint deviation from M to M' is Pareto improvement within the deviating coalition.

We must also show that this deviation is stable. Suppose not. Then there exists a (strict) sub-coalition $C \subset M' \setminus M$ that would deviate back to voting sincerely. It must be that C contains at least two judges, since otherwise it is a unilateral deviation, which cannot be, since M' is a Nash equilibrium coalition. (This implies that $M' \setminus M$ contains at least 3 judges.) Take some $k \in C$. By construction, $M' \setminus C \subset M' \setminus \{k\} \subset M'$, and so $\gamma(M' \setminus C) \leq \gamma(M' \setminus \{k\}) \leq \gamma(M')$. Since the deviation from the deviation is profitable, we have: $u_P(\gamma(M' \setminus C), x^k) > u_P(\gamma(M', x^k)) + \alpha l(z, x^k) \geq u_P(\gamma(M' \setminus \{k\}), x^k)$, where the second inequality follows from the fact that M' is an equilibrium coalition. Hence $u_P(\gamma(M' \setminus C), x^k) > u_P(\gamma(M' \setminus \{k\}), x^k)$, which cannot be since $\gamma(M' \setminus C) \leq \gamma(M' \setminus \{k\}) < x^k$. Hence, the deviation is stable. □

Lemma 5. *Let (d, M) be an adjudication (Nash) equilibrium. There exists a connected coalition M' with $|M'| = |M|$ and such that (d, M') is also an adjudication (Nash) equilibrium coalition. Moreover, (d, M') can be sustained as an adjudication equilibrium over a (weakly) larger range of values of α than (d, M).*

__Proof of Lemma 5__. Let (d, M) be an adjudication (Nash) equilibrium, and suppose M is not connected. WLOG, suppose $d = 1$, so that, by Lemma 2, $M^1(z) \subset M$. Since M^1 is a connected coalition and M is disconnected, M must contain members of $M^0(z)$. Then there exists $i < j$ with $i, j \in M^0(z)$, $i \notin M$ and $j \in M$. Then $z < x^i \leq x^j$. Let M' be identical to M except that judge j is replaced by judge i. By part 2 of Assumption 1, it must be that $\gamma(M) = \gamma(M')$. (To see this, note that replacing judge i with j causes the social acceptance set to be unchanged, since both judges will make the same proposal $\overline{y}$.) Since M is an equilibrium, it must be that:

$$u_P(\gamma(M), x^j) + \alpha l(z - x^j) \geq u_P(\gamma(M - \{j\}), x^j) \tag{6}$$

and:

$$u_P(\gamma(M \cup \{i\}), x^i) + \alpha l(z - x^i) < u_P(\gamma(M), x^i) \tag{7}$$

We seek to show that M' is also an equilibrium coalition. It suffices to show that:

$$u_P(\gamma(M'), x^i) + \alpha l(z - x^i) \geq u_P(\gamma(M' - \{i\}), x^i) \tag{8}$$

and:

$$u_P(\gamma(M' \cup \{j\}), x^j) + \alpha l(z - x^j) < u_P(\gamma(M'), x^j) \tag{9}$$

If $x^i = x^j$, it is trivial to do so, since i and j have identical preferences. Suppose $x^i < x^j$. Note that:

$$\begin{aligned}
&\left\{\left[u_P(\gamma(M), x^j) - u_P(\gamma(M - \{j\}), x^j)\right] + \alpha l(z - x^j)\right\} - \left\{\left[u_P(\gamma(M'), x^i) - u_P(\gamma(M' - \{i\}), x^i)\right] + \alpha l(z - x^i)\right\} \\
&= \left(\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x^j)dy + \alpha l(z - x^j)\right) - \left(\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x^i)dy + \alpha l(z - x^i)\right) \\
&= \int_{x^i}^{x^j} \frac{\partial}{\partial x}\left[\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x)dy + \alpha l(z - x)\right] dx \\
&= -\int_{x^i}^{x^j}\left[\int_{\gamma(M-\{j\})}^{\gamma(M)} l'(y - x)dy + \alpha l'(z - x)\right] dx \\
&\leq 0
\end{aligned}$$

where the final line follows from the fact that $\gamma(M - \{j\}) < \gamma(M) \leq z < x^i < x^j$ and that, by the IDID property, $l'(y - x) > 0$ for all $y < x$. It follows that (6) implies (8). By a similar argument, we can show that (7) implies (9). Hence, M' is an equilibrium coalition as well.

Moreover, if $x^i < x^j$, then the inequality above is strict, and continues to be so for some $\alpha' > \alpha$ and even for some $\gamma(M') < \gamma(M)$. □

Proof of Proposition 3. The existence of an adjudication (Nash) equilibrium follows by standard game theoretic results. We now establish the existence of a CCPAE. Let (d_0, M_0) be an adjudication (Nash) equilibrium, and suppose it is a candidate to be a CCPAE. By Lemma 4, we know that there is no larger adjudication equilibrium with the same case disposition (i.e. there is no M' with $M_0 \subset M'$ s.t. (d_0, M') is an adjudication equilibrium). If (d_0, M_0) is not a CCPAE, then there must exist some other coalition C_0 and induced disposition d_0' s.t. all the members of $M_0 \cap C_0$ prefer to deviate from (d_0, M_0) to (d_0', C_0). Moreover, no subset of the deviators $M_0 \cap C_0$ should have a strict incentive to deviate from C_0. Immediately, this implies that (d_0', C_0) is an adjudication (Nash) equilibrium.

By construction, it cannot be that $d_0' = d_0$, since any smaller coalition inducing the same case disposition must be inferior for the deviating judges (by Lemma 4). Hence $d_0' = 1 - d_0$. Using the same logic as in Lemma 5, if C_0 is disconnected, we can always find some other coalition C_0' that is connected and which implies a strictly favorable deviation for the judges in $M_0 \cap C_0'$. Hence, it is WLOG to focus on deviations by connected coalitions. Hence (d_1, C_0) is a connected adjudication (Nash) equilibrium, where $d_1 = 1 - d_0$. Let (d_1, M_1) be the largest connected coalition that implements case disposition $d_1 = 1 - d_0$. Clearly $C_0 \subseteq M_1$. (d_1, M_1) is the only other candidate for a CCPAE. Suppose it is not. Then, by the same argument, there must be some connected $C_1 \subseteq M_0$, s.t. (d_0, C_1) is preferred by all judges in the deviating coalition $M_1 \cap C_1$, and this deviating coalition is stable.

Since each deviation flips the case disposition, and coalitions are connected, then the median judge must be a member of the deviating coalition in each case. WLOG, suppose $d_0 = 0$ and $d_1 = 1$. We have:

$$u_P(\gamma(C_0), x^{med}) + \mathbf{1}[z < x^{med}]l(z - x^{med}) > u_P(\gamma(M_0), x^{med}) + \mathbf{1}[z > x^{med}]l(z - x^{med}) \quad (10)$$

and

$$u_P(\gamma(C_1), x^{med}) + \mathbf{1}[z > x^{med}]l(z - x^{med}) > u_P(\gamma(M_1), x^{med}) + \mathbf{1}[z < x^{med}]l(z - x^{med}) \quad (11)$$

Suppose $x^{med} < z$. By assumption 1, $\gamma(C_0) \leq \gamma(M_1) \leq z \leq \gamma(M_0) \leq \gamma(C_1)$. It cannot be that $x^{med} \leq \gamma(M_1)$, otherwise $u_P(\gamma(M_1), x^{med}) > u_P(\gamma(C_1), x^{med})$, which contradicts (10). Hence: $\gamma(C_0) \leq \gamma(M_1) < x^{med} < z \leq \gamma(M_0) \leq \gamma(C_1)$. But then, by the strict quasi-concavity of u_P, $u_P(\gamma(M_0), x^{med}) \geq u_P(\gamma(C_1), x^{med}) > u_P(\gamma(M_1), x^{med}) \geq u_P(\gamma(C_0), x^{med})$. But (10) implies that $u_P(\gamma(C_0), x^{med}) > u_P(\gamma(M_0), x^{med})$. We have a contradiction. By a symmetric argument, we can show that a contradiction arises in the scenario that $x^{med} > z$. Hence, it cannot be that both (d_0, M_0) and (d_1, M_1) are both *not* CCPAE. Existence is established.

Establishing the equilibrium properties is straight-forward. Fix a case z. Suppose (d, M) is a CCPAE. By Lemma 2, $M^d \in M$. Suppose $M^d \neq \emptyset$. Then, by the ordering over judges, $1 \in M^d$ if $d = 1$ and $n \in M^d$ if $d = 0$. Since M is connected and contains at least $k = \frac{n+1}{2}$ agents, then $\frac{n+1}{2} \in M$. Hence either $\{1, ..., \frac{n+1}{2}\} \subset M$ or $\{\frac{n+1}{2}, ..., n\} \subset M$. (If $M^d = \emptyset$, then the result follows provided that we rule out equilibria that relies upon a majority of judges voting strategically, but not those judges with the lowest cost of doing so.) □

Proof of Corollary 1. To show part (1), let (d, M) and (d', M') be distinct CCPAE, and suppose that $d = d'$. Then, by Lemma 2, $M^d(z) \subset M$ and $M^{d'}(z) \subset M'$. Since M and M' are connected, this implies (WLOG) that $M \subset M'$. But then, by Lemma 4, M cannot be coalition-proof, which is a contradiction. Hence, $d \neq d'$. Since distinct CCPAE must have distinct dispositions, and there are only two possible dispositional values, then there can be at most two CCPAE.

Next, we establish part (2). Fix some case z. For $j = \{1, .., \frac{n-1}{2}\}$, define:

$$\alpha_j(z) = \frac{u_P(\gamma(\{j+1, \dots, n\}), x^j) - u_P(\gamma(\{j, \dots, n\}), x^j)}{l(z - x^j)}$$

If $x^j < z$, so that j's ideal disposition is $d = 1$, then whenever $\alpha > \alpha_j(z)$, there cannot be an adjudication equilibrium in which j is the left-most judge who votes strategically. Similarly, for $j = \{\frac{n+3}{2}, \dots, n\}$ define:

$$\alpha_j(z) = \frac{u_P(\gamma(\{1,, \dots, j-1\}), x^j) - u_P(\gamma(\{1, \dots, j\}), x^j)}{l(z - x^j)}$$

If $x^j > z$, so that j's ideal disposition $d = 0$, then whenever $\alpha > \alpha_j(z)$, there cannot be an adjudication equilibrium in which j is the right-most judge who votes strategically. Finally, define:

$$\alpha_{\frac{n+1}{2}}(z) = \frac{u_P(\gamma(\{1, \dots, \frac{n+1}{2}\}), x^{med}) - u_P(\gamma(\{\frac{n+1}{2}, \dots, n\}), x^{med})}{l(z - x^{med}})$$

Recall $M^1(z)$ and $M^0(z)$ are the coalitions that arise if judges vote sincerely. Since n is odd, one of these will be larger than the other. We refer to the larger coalition as the 'sincere majority coalition' and the smaller coalition as the 'sincere minority coalition'.

We consider two scenarios. First, suppose $|M^1(z)| - |M^0(z)| \geq 2$. This implies that if judges vote sincerely, the size of the majority and minority coalitions will differ by at least two. Then, for all $\alpha \geq 0$, there exists an adjudication (Nash) equilibrium in which all members of the sincere majority coalition vote sincerely. (To see why, note that if all judges in the sincere majority coalition vote sincerely, then no judge is pivotal over the case disposition. The result is then an immediate consequence of Lemma 2. Note, of course, that judges in the sincere minority might nevertheless have an incentive to vote strategically.)

We show that, for α sufficiently large, there cannot be an adjudication (Nash) equilibrium which implements the opposite disposition. Suppose there is. By Lemma 5, we know that it suffices to focus on connected equilibria. Suppose $M^1(z) > M^0(z) + 1$, so that the sincere disposition is $d = 1$. The connected majority coalitions that implement the opposite disposition ($d = 0$) and satisfy Lemma 2 are of the form: $\{j, , , n\}$, where $j \in \{1, .., \frac{n+1}{2}\} \subseteq M^1(z)$. Define $\alpha(z) = \max\{\alpha_1, ..., \alpha_{\frac{n+1}{2}}\}$. By construction, if $\alpha > \alpha(z)$, then none of these coalitions is consistent with an adjudication equilibrium. Hence, if $\alpha > \alpha(z)$, there cannot be any adjudication equilibria that implement the sincere minority's preferred disposition. Hence, any adjudication equilibrium must implement the sincere majority's preferred disposition. By previous arguments, there is a unique CCPAE that achieves this.

Suppose instead that $M^0(z) > M^1(z) + 1$, so that the sincere disposition is $d = 0$. Then the result obtains by defining $\alpha(z) = \max\{\alpha_{\frac{n+1}{2}}, ..., n\}$.

Next, consider the scenario where $|M^1(z) - M^0(z)| = 1$, so that, if all judges vote sincerely, the median is pivotal. This scenario differs from the previous one only insofar as the median judge may have an incentive to vote strategically for α low enough, even if all other judges in the sincere majority vote sincerely. Again, first suppose that $x^{med} < z$, so that the sincere disposition is $d = 1$. Define:

$$\alpha(z) = \min\left\{\max\{\alpha_1, ..., \alpha_{\frac{n+1}{2}}\}, \max\{\alpha_{\frac{n+3}{2}}, ..., \alpha_n\}\right\}$$

Following the same logic, there is a unique equilibrium provided that $\alpha > \alpha(z)$. Supposing instead that $x^{med} > z$, then the result obtains by defining:

$$\alpha(z) = \min\left\{\max\{\alpha_1, ..., \alpha_{\frac{n-1}{2}}\}, \max\{\alpha_{\frac{n+1}{2}}, ..., \alpha_n\}\right\}$$

□

Proof of Lemma 3. Follows immediately from the proofs of Proposition 3 and Corollary 1. □