Appendices

C Endogenous Labor Supply (Online Appendix)

In this section, we consider a micro-founded version of the model, analogous to **?**, in which agents supply labor elastically, and dead-weight losses arise endogenously as a consequence of labor market distortions. In doing so, we note that micro-foundations will affect the results only insofar as they change salient properties of the indirect utility function $v(\tau, x)$. Absent such changes, all of the results from sections 3 and 4 will continue to hold. Hence, it suffices to check the properties of $v(\tau, x)$.

There is a unit mass of agents. Each agent is characterized by their productivity x which is an i.i.d. draw from some distribution F. From herein, we refer to an agent's productivity as their type. Agents have preferences $u(c) + w(l)$ defined separably over consumption c and leisure l. Utility is increasing in both consumption and leisure ($u' > 0$ and $w' > 0$), and u and w both are concave, with at least one strictly concave. Agents are endowed with one unit of time, which they may allocate between leisure and work effort n. For simplicity, we assume $\lim_{l \to 0} w'(l) = \infty$, which rules out the corner solution in which some agent spends all of her time working. Agents supply their labor in competitive labor markets, and earn a wage equal to their productivity. Hence, the income of an agent with productivity x is $y = xn$.

The government levies a proportional tax τ on labor income that finances a lump-sum transfer T to each agent. Given the government policy (τ, T), the consumption of a type-x agent is: $c = T + (1-\tau)\,xn$. The agent's problem is to choose the quantity of labor to supply to maximize:

$$\max_{n \in [0,1]} u\left(T + (1-\tau)\,xn\right) + w\left(1-n\right)$$

Given that preferences are strictly concave, the problem has a unique maximizer $\hat{n}\left(\tau, T; x\right)$. The maximizer is the solution to the first order condition:

$$(1-\tau)\,xu'\left(T + (1-\tau)\,xn\right) - w'\left(1-n\right) \leq 0$$

with strict equality unless $\hat{n} = 0$. This will occur if $(1-\tau)\,xu'\left(T\right) - w\left(1\right) < 0$, which implies that:

$$x < \frac{1}{1-\tau} \cdot \frac{w'\left(1\right)}{u'\left(T\right)} = x_0\left(\tau, T\right)$$

Hence, all but the least productive agents will work. It is easily verified that work-effort is decreasing in the size of the transfer (i.e. $\frac{\partial \hat{n}}{\partial T} \leq 0$, with strict inequality whenever $x > x_0$[1]),

[1] To see this, let $D = (1-\tau)^2\, x^2 u''\left(\hat{c}\right) + w''\left(\hat{l}\right)$. By the strict concavity of preferences, $D < 0$. Applying the implicit function theorem to the first order condition gives: $\frac{\partial n}{\partial T} = \frac{(1-\tau)xu''(\hat{c})}{-D} < 0$.

which implies that leisure is a normal good. Let $\hat{y}(\tau, T; x) = x\hat{n}(\tau, T; x)$ denote the income of a type-x agent. Notice that $\frac{\partial \hat{y}}{\partial x} = \frac{(1-\tau)xu'(\hat{c}) - nw''(\hat{l})}{-D} > 0$, and so agents' incomes are monotone in their productivity.

The average income in the economy is: $\overline{y}(\tau, T) = \int_0^\infty \hat{y}(\tau, T; x)dF(x)$. Since the government policy must be feasible, we have $T = \tau\overline{y}(\tau, T)$. Intuitively, the government budget constraint establishes a feasible level of transfers $T(\tau)$ for each level of taxes τ.[2] Hence, the government's redistribution policy amounts to the choice of a tax rate τ. Moreover, we assume that households understand that government policy is subject to its budget constraint; there is no fiscal illusion. Accordingly, let $n(\tau; x) = \hat{n}(\tau, T(\tau); x)$ and $y(\tau; x) = \hat{y}(\tau, T(\tau); x)$ be the labor supply and income of a type-x agent, given tax rate τ and the associated transfer $T(\tau)$. Similarly, let $\overline{y}(\tau) = \int_0^\infty y(\tau; x)dF(x)$ denote the average income, given tax rate τ and the associated transfer $T(\tau)$.

Let $v(\tau, x)$ denote the indirect utility function of a type x agent. We have:

$$v(\tau, x) = u(\tau\overline{y}(\tau) + (1-\tau)xn(\tau, x)) + w(1 - n(\tau, x))$$

We seek to establish the parallels between the properties of the indirect utility functions from the structural and reduced-form approaches. By the envelope theorem, $v_\tau(\tau, x) = \left[\frac{\partial \tau\overline{y}}{\partial \tau} - y(\tau; x)\right] u'(c(\tau, x))$. This is directly analogous to the correponding expression (equation **??**) in the reduced-form model.[3]

The indirect utility function does not generically inherit the curvature properties of the direct utility function. In particular, v need not be concave in τ. However, we have the following result:

Lemma C1. *The indirect utility function $v(\tau, x)$ is pseudo-concave in τ for each x.*

The pseudo-concavity of $v(\tau, x)$ guarantees that the first order conditions characterize the optimal tax rate for each voter. Analogous to the reduced-form case, any voter whose income would be above average when there is zero taxation will prefer zero taxation, and all other agents will demand a positive level of redistribution, with the optimal tax rate satisfying: $\overline{y} + \tau\frac{\partial \overline{y}}{\partial \tau} - y(\tau; x) = 0$. This condition is identical to equation (13) in **?**, which defines the optimal tax rate in their framework.[4]

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[2] To see this formally, fix any $\tau \in [0, 1]$. Define the function $\psi(T; \tau) = \tau\overline{y}(\tau, T) - T$. Notice that $\psi(0; \tau) > 0$ and $\lim_{T\to\infty} \psi(T; \tau) < 0$ and $\frac{\partial \psi}{\partial T} = \tau\frac{\partial \overline{y}}{\partial \tau} - 1 < 0$, since $\frac{d\overline{y}}{\partial T} < 0$. The result follows by the intermediate value theorem.

[3] In fact, if the reduced-form function $e(\tau)$ satisfies $e'(\tau) = 1 + \frac{\partial \overline{y}}{\partial \tau} \cdot \frac{\tau}{\overline{y}} = 1 + \varepsilon(\tau)$, where $\varepsilon(\tau)$ is the average tax elasticity of labor supply, then the expressions for the marginal utility of taxation are identical across the two models. For example, if preferences are given by $c - \frac{\theta}{1+\theta}(1-l)^{\frac{1+\theta}{\theta}}$ (as in **?** and **?**, amongst others) then the tax elasticity of labor supply is a constant $\varepsilon(\tau) = -\theta$, and so the implied reduced-form dead-weight loss function is $e(\tau) = (1+\tau)\theta + \theta\ln(1-\tau)$.

[4] Pseudo-concavity does not guarantee that the problem admits a unique maximizer, although the set of optimizers is guaranteed to be convex. We follow **?** in assuming a unique solution.

Finally, we establish that a condition, analogous to Assumption 1 ensures that agents' preferences over tax policies satisfy the Spence-Mirrlees condition. First note that:

$$v_{\tau x} = \left[(1-\tau) \left(\frac{\partial \tau \overline{y}}{\partial \tau} - y(\tau, x) \right) u''(c(\tau, x)) - u'(c(\tau, x)) \right] \frac{\partial y(\tau, x)}{\partial x}$$

Since $\frac{\partial y(\tau,x)}{\partial x} > 0$, the sign of $v_{\tau x}$ depends on the sign of the term in square brackets. Now, as in the reduced-form case, the condition is guaranteed to be satisfied for agents with incomes that are sufficiently low (i.e. if $y(\tau, x) < \frac{\partial \tau \overline{y}}{\partial \tau}$). For agents with larger incomes, the Spence-Mirrlees condition is satisfied provided that:

$$R(c^*) < \frac{(1-\tau)\, y(\tau, x) + \tau \overline{y}\,(\tau)}{(1-\tau)\, y^{(}\tau, x) - (1-\tau) \frac{\partial (\tau y)}{\partial \tau}}$$

which is analogous to Assumption 1. The marginal utility of taxation is monotone in agents' incomes provided that the coefficient of relative risk aversion is not too large for high productivity agents. It is easily shown that this assumption is equivalent to the assuming that $\frac{\partial n}{\partial \tau} < 0$ for all τ and all x.[5]

As we briefly noted in section 2, when labor supply is elastic, imposing the Spence-Mirrlees condition is equivalent to assuming that taxation deters work effort. To make sense of this, note that increasing labor taxes (whilst simultaneously increasing transfers) has two effects. The substitution effect unambiguously deters work effort, whilst the sign of the wealth effect is ambiguous. Since a tax increase is combined with an increase in transfers, the wealth effect (further) deters work effort whenever $y(\tau, x) < \frac{\partial \tau \overline{y}}{\partial \tau}$, and stimulates it otherwise.[6] The Spence Mirrlees condition implies that, for high productivity agents, the wealth effect cannot be so large as to overwhelm the substitution effect. Following an increase in taxes, all agents work less.

Thus, we have shown, with the exception of concavity, all salient features of the indirect utility function are implied by micro-foundations. The fact that $v(\tau, x)$ is not guaranteed to be strictly concave is unfortunate, but not fatal. We have already shown that the optimal tax rate is characterized by the first order conditions, notwithstanding the failure of concavity. The other main role played by concavity was in guaranteeing that the bargaining game admitted an equilibrium in no delay.[7] To achieve this result, concavity is sufficient, but not necessary. No-delay equilibria require that, for every agent, there is *some* decisive coalition C including that agent, for which the associated coalitional acceptance set A_C is non-empty.

[5] To see this, note that $\frac{\partial n}{\partial \tau} = x \frac{\left[(1-\tau)\left(\overline{y} + \tau \frac{\partial \overline{y}}{\partial \tau} - y(\tau,x)\right)u''(c^*) - u'(c^*)\right]}{-D}$. Hence $v_{\tau x} = \frac{(-D)}{x} \cdot \frac{\partial y}{\partial x} \cdot \frac{\partial n}{\partial \tau}$. Then since $D < 0$ and $\frac{\partial y}{\partial x} > 0$, then $sign\,(v_{\tau x}) = sign\left(\frac{\partial n}{\partial \tau}\right)$.

[6] To see this, first note that absent the increase in transfers, the Slutsky Equation implies: $\frac{\partial n}{\partial \tau} = \frac{\partial n^h}{\partial \tau} - y \frac{\partial n}{\partial T}$. Adding the increased transfers gives an overall effect: $\frac{\partial n^h}{\partial \tau} - \left(y - \frac{\partial \tau \overline{y}}{\partial \tau}\right) \frac{\partial n^*}{\partial T}$.

[7] We also used concavity to establish that the social acceptance set was an interval bounded by the certainty equivalents of the left and right decisive voters. Absent concavity, the social acceptance set is not guaranteed to be an interval. However, the boundaries of the convex hull of the acceptance set will continue to be the respective certainty equivalents, and these will continue to converge to the same limit as in Proposition **??**.

Concavity of preferences guaranteed that the acceptance set for *every* coalition is non-empty – which is clearly stronger that necessary. It suffices, for example, that the acceptance set associated with the smallest connected coalition containing a given agent be non-empty. With continuous preferences, this may plausibly be the case, even when some agents' preferences are non-concave, provided that they are not too convex.

Proof of Lemma C1. To prove the pseudo-concavity of the indirect utility function it suffices to show that, for $\tau \in (0,1)$, if $v_\tau(\tau^*, x) = 0$ then v_τ achieves a maximum at τ^*. Suppose $v_\tau(\tau^*, x) = 0$ for some $x > 0$. Recall $v_\tau(\tau, x) = \left[\frac{\partial \tau \bar{y}}{\partial \tau} - y(\tau; x)\right] u'\left(c(\tau, x)\right)$ and that income y is monotonically increasing in productivity x. Thus, whenever $x' < x$, then $v_\tau(\tau^*, x') > 0$. Similarly, whenever $x'' > x$, $v_\tau(\tau^*, x'') < 0$.

Takes some small $\varepsilon > 0$. Since v is continuously differentiable, it follows that $v_\tau(\tau^* - \varepsilon, x') > 0$ for ever $x' < x$. By continuity, this implies that $v_\tau(\tau^* - \varepsilon, x) \geq 0$. Similarly, $v_\tau(\tau^* + \varepsilon, x) \leq 0$. Together, these rule out τ^* as a minimizer (which would require $v_\tau(\tau^* - \varepsilon, x) < 0$) or as a saddle point (which would require that $v_\tau(\tau^* - \varepsilon, x)$ and $v_\tau(\tau^* + \varepsilon, x)$ have the same sign. $\square$