The Evolution of the Common Law with Strategic Litigants*

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Abstract

The common law is shaped by the cases that are litigated in court. We study the incentives for litigants to influence legal evolution by strategically choosing which disputes to litigate. In our framework, clarifying the law typically benefits defendants. This creates a strict incentive for plaintiffs to settle cases, or to abandon legal claims even when litigation is costless. When plaintiffs are regulators, we associate this scenario with 'regulator capture'. By contrast, defendants have an incentive to force litigation, even in instances where plaintiffs would ordinarily not litigate, by generating 'test cases'. We predict that settlement and regulatory capture is most likely when regulators are sufficiently long-run oriented, whilst test cases arise when defendants are. We analyze the welfare consequences arising from these dynamic incentives.

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1 Introduction

Within the common law tradition, courts are largely reactive institutions. Typically, they do not "seek out specific interpretive occasions, but instead wait for others to bring matters to their attention" (Fiss, 1984). Accordingly, "judge made" law can be viewed as a function of the demand for judicial decisions by plaintiffs. Thus, the trajectory of how a particular law evolves will depend on the "stream" of (relevant) cases that are brought before the court by plaintiffs (Zywicki, 2012).

In the modern state, these plaintiffs are often regulatory agencies with strategic concerns regarding the types of cases that they choose to litigate. For example, the Securities and Exchange Commission's (SEC) or the Federal Trade Commission (FTC) have, over decades, prosecuted various firms for violating regulations within their respective purviews. For a regulator taking a long-run view, the decision of whether to initiate litigation, and whether to offer a firm a settlement, will reflect the regulator's concerns, not only about outcomes in the instant case, but of how future outcomes are affected. Thus, regulators may have an incentive to strategically manipulate current enforcement actions to affect legal precedent and, by extension, the court's decisions concerning *future* enforcement actions.

Besides regulators, plaintiffs may also be special interest groups that take up either class action law suites on behalf of the victims or other constitutes. Well-known examples of such litigants are labor unions and the American Civil Liberties Union which have played an important role in litigating on behalf of workers and civil rights. Likewise, de Poorter (Depoorter, 2013) notes that the National Rifle Association "has a long tradition of financially supporting selected disputes in litigation" in order to advocate on behalf of gun-owners. Such long lived plaintiffs may selectively decide which cases to settle pre-trial and which cases to pursue in court, in order to influence the precedent, which will in turn impact the outcomes of future cases that they may litigate.

Potential defendants can also affect the tragectory of the common law by inviting litigation. For example, in the runup to the 2008 election Citizens United (a conservative non-profit) invited litigation from a regulator, the U.S. Federal Election Commission (FEC), by creating a documentary criticizing Hillary Clinton. In doing so, they invited the plaintiff (the FEC) to charge them with violating the Bipartisan Campaign Reform Act (BRCA, popularly known as the McCainFeingold Act) which prohibits "electioneering communications" by incorporated entities. The litigation that Citizens United invited upon themselves led to the eponymous landmark ruling in *Citizens United v. Federal Election Commission*.[1] Similarly, Arizona's immigration law SB 1070, was crafted in a way to provoke legal challenges in order to have the supreme court clarify the limits of federal authority concerning immigration *Arizona v. United States*.[2] Test cases also arise in the area of intellectual property law and patent protection, wherein firms patent a design that may invite litigation from competitors in order to determine whether such aspects of a product's design are patentable *Apple Inc. v. Samsung Electronics Co., Ltd.*.[3] In these instances firms or other entities may "push the legal envelop" in order to invite litigation and in doing so force the courts to clarify the law, thereby shaping its evolution.

In this paper, we explore these avenues for plaintiffs and defendants to strategically manipulate the common law's evolution to their respective advantage. Specifically, whether it is plaintiffs or defendants who have incentive to settle out of court to avoid having legal precedent established. And, whether plaintiffs or defendants have incentive to generate test cases to "force the law" to evolve to their benefit. Thus, in a very broad sense we wish to understand how actors can affect the path of the law, rather than passively respond or react to the law.

This investigation must be contrasted with the extant literature in regulation, in which it is generally assumed that legal actors take the law as "given," and respond accordingly. Indeed,

[1] *Citizens United v. Federal Election Commission*, 558 U.S. 310 (2010).
[2] *Arizona v. United States*, 567 U.S. 387 (2012).
[3] *Apple Inc. v. Samsung Electronics Co., Ltd.*, 137 S. Ct. 429 (2016).

Galanter (1974) notes that most analyses of the legal system "start at the rules end... and then see what affect [they] have on the parties." The decision to settle provides a case in point: legal actors can decide whether or not to settle out of court depending on whether they seek to establish or avoid setting a precedent by going to trial. This incentive will be especially strong for repeat players. As Galanter (1974) notes: "repeat players can play for rules as well as immediate gains,...". That is, repeat players make decisions that not only affect their immediate outcomes but also try to shape the law to improve their future prospects and payoffs. Since regulators (and other interest groups) are often repeat players, the decision to settle will not depend solely on the current net benefits relative to going to trial, but also on how the court's ruling (or lack thereof) affects their future ability to regulate firms. Accordingly, our paper extends the extant literature by formalizing Galanter (1974) to understand precisely how the concerns of long lived litigants affect the common law's evolution. Therefore, we examine whether and which plaintiffs have incentives to settle when doing so prevents the law from being clarified, and how anticipation of these incentives by the defendant affects their own decision-making, especially with respect to the intensity of conduct that may harm third parties.

To investigate these issues we build on previous models that capture the evolutionary process of the common law tradition (see Baker and Mezzetti, 2012; Niblett, 2013; Parameswaran, 2018). A firm's production generates a negative externality. The marginal social cost of this externality (and thus the socially efficient output level) is uncertain, but its distribution is common knowledge. Given this uncertainty, the firm's output will fall into one of three possible categories: it may either be definitely below the socially efficient level, definitely above the socially efficient level, or there may be ambiguity about whether the output is above of below the efficient level. A welfare-maximizing court implements a negligence-type rule, in which the firm is definitely not held liable in the first case ('*per se immunity*'), and definitely held liable in the second ('*strict liability*'). The legal rule is silent in the region of ambiguity. If a case arises in this region, an evidentiary trial may be held if a plaintiff brings the case to trial.

Plaintiffs in our framework are regulators or other special interest litigants who act as agents of the victims and may seek legal action against the firm. Since cases in either the region of immunity or strict liability are automatically decided, the decision to seek legal action is critical only when confronted with a case in the ambiguous region. When confronted with a case in the ambiguous region, the plaintiff can do one of three things: nothing, settle, or take the firm to court. Thus, it is only in this region, and then only if the plaintiff brings the case, that the court's role is salient.

The above framework introduces a critical, relatively novel, feature of our model; namely, that enforcement is *jointly undertaken by both the court and the regulator*. If the size of the externality were known, then the regulator's decision would amount to imposing a "Pigovian tax" on firms that are liable, or implementing the optimal "Beckerian" (second-best) policy. However, because the cost is unknown, the regulator can sanction firms that produce in the ambiguous region *only if the court* finds them liable upon trial and investigation. Thus, the regulator depends on the court to determine whether the firm is liable. Conversely, the court is dependent on the regulator for its stream of cases. While the regulator cannot sanction a firm without the court, the court can only decide upon cases that are brought before it by the regulator. In this regard our paper departments from the extant literature (Baker and Mezzetti, 2012; Parameswaran, 2018) in that enforcement is conducted jointly by two strategic parties and the stream of cases brought before the court is not automatic but rather determined by the strategic interests of the plaintiff.

To establish similarities and differences with previous models, we begin by establishing baseline outcomes in a static model where there is no learning motive. We show that the firm will either produce the largest output that is guaranteed to not attract litigation (the 'safe output'), or the output in the ambiguous region that optimally trades-off higher profits against the penalty if held liable by the court. The 'risky output' is independent of litigation costs, and is unaffected by whether settlement is anticipated or not.

We then consider a two period game with a long lived plaintiffs and defendants. We find that the plaintiff is willing to accept settlements that are smaller than the settlement amount in the static game *even in the absence of any direct court costs*. Why? Clarifying the law makes the plaintiff strictly worse off. Intuitively, with more information, the firm can make choices that enable it to maximize profits at lower risk of being held liable. These choices may still entail significant harms to the victim, although they are now more likely to go uncompensated. As a result the plaintiff is willing to accept settlements that are smaller than those in the static game.

For analogous reasons the defendant prefers to clarify the law by going to trial because it enables it to produce the maximally safe quantity with impunity. How these two effects influence the common law's evolution depends on the patience of the litigants. First, if the defendant is sufficiently patient relative to the plaintiff, then they choose a quantity that is sufficiently high in order to "provoke" litigation. That is, they generate test cases which force learning. Second, when defendants are relatively impatient compared to plaintiffs, cases usually settle. However, even here because plaintiffs prefer to avoid trial, the quantity chosen by the defendant is usually higher (and never lower) than what it would have been in a static game.

Our model also enables us to analyze the welfare implications of the parties' choices. We identify two dimensions of efficiency: 'static efficiency', which measures the firm's output choice in each period relative to the stage game benchmark ideal; and 'dynamic efficiency', which measures the extent to which the litigants choices result in greater clarity in the law (which in turn incentivizes more statically efficient decision making in future periods). We show that how well the equilibrium performs along each criterion is strongly affected by the preferences and degree of long-run orientation of both the plaintiff and defendant.

Other dynamic models of judicial decision-making investigate how the common law evolves, and its the implications for efficiency. But their focus is on the understanding the welfare implications when judges (or courts) have heterogeneous preferences (see Gennaioli and Shleifer (2007), Ponzetto and Fernandez (2008), and Niblett (2013)). For example, Gennaioli and Shleifer (2007) provide foundations for the Cardozo Theorem, which states that the individual biases of judges tend to wash out as law is created piece-meal, and that legal evolution is, on average, efficiency enhancing. Ponzetto and Fernandez (2008) similarly show that the common law converges towards more efficient rules, making it more effective in the long-run than statutory rule making. These models typically do not involve any uncertainty about the ideal legal rule although the game is dynamic, there is no role for learning. In a similar vein, Micelli (2010) studies how the common law evolves in the specific context of strict liability and negligence in situations where one liability rule is sometimes more efficient for society. He assumes that judges may be biased in favor of (or against) one form of liability. Within this context he finds that the law does not evolve to the efficient legal rule unless judges are efficiency minded (and unbiased). Indeed, in his framework biased judges may prevent the law from evolving. While in that regard his paper is similar to ours, the core focus is whether judges will allow the law to evolve. Whereas, in our paper judges do not play a significant role. Rather, although judges *are* efficiency minded, the law does not evolve because of the strategies of the litigants who choose which cases to bring to trial.

Accordingly, this article contributes to the literature on the efficiency of common law more broadly. In a seminal article, Posner (1973) argued that the decisions of efficiency-minded judges would tend to cause the common law to produce efficient rules. Subsequent articles have examined why the common law may tend to efficiency, even if judges are imperfect in their motives, information or execution. Priest (1977) and Rubin (1977) provide a 'demand-side' argument, that inefficient legal rules are more likely to be litigated and subsequently overturned, than efficient ones. Cooter and Kornhauser (1980) provide a formalization of these arguments. By contrast, Hylton (2006) shows that information asymmetries between litigants can cause biased rules, which favor the more informed party, to evolve. Zywicki

(2002) stresses the importance of 'supply-side' factors, such as institutional rules and norms, in determining the efficiency of common law. Cooter et al. (1979) show that incremental rule-making by courts can converge upon efficient rules, even if individual courts are imperfectly informed about the ideal rule. Hadfield (2011) asks whether legal rules will dynamically adapt to new or local conditions. By and large, the primary force that drives the law, in these articles, is case selection (i.e. which cases are litigated). The opposite effect, of the law driving the sorts of cases that arise, is typically left unexplored (although, see Png (1987) and Parameswaran (2018)).

Besides contributing to the efficiency debate surrounding the common law, this paper is also related to the growing literature that studies the role of learning in judicial decision-making (see, in particular, Hadfield (1991), Baker and Mezzetti (2012), Fox and Vanberg (2014), Callander and Clark (2017), Miceli (2010), and Parameswaran (2018)). Indeed, our model setup is identical to Parameswaran (2018), except that that paper abstracts from the possibility of settlement. All of the above papers focus on dynamic decision making by an informationally constrained court that learns as it hears cases. A particular interest of these papers is whether courts construe rules broadly versus narrowly, and the implications of these choices for efficiency of the common law. Baker and Mezzetti (2012) and Callander and Clark (2017). In Baker and Mezzetti (2012), the court trades off the benefit of learning from cases against the explicit adjudication costs. When confronted with a case whose ideal disposition is unknown, the court summarily dispose of the case if it is 'close enough' to an existing case with known disposition, and investigate the case otherwise. The court, thus implements, 'somewhat broad' legal rules. Baker and Mezzetti (2012) show that, if adjudication costs are not too high, the law will evolve incrementally and eventually converge to the ideal legal rule. Parameswaran (2018) extends this analysis to explicitly include the agent behavior that generates cases. He shows that with legal uncertainty, the stream of outputs chosen by the firm will be biased away from the *ex ante* efficient level, which skews the court's ability to learn.[4] In effect, the adjudication costs in Baker and Mezzetti are replaced with the implicit (bias) costs stemming from endogenous response to legal rules. Parameswaran (2018) shows that the size of this implicit (bias) cost is itself endogenous and increases as the law evolves, creating a dynamic where the law evolves for a finite period before settling. Moreover, legal evolution stops before the court can implement the ideal legal rule.

Our model differs from each of these in that, whereas they are particularly interested in optimal decision making by the court, we consider an essentially passive court. Instead, we focus on the incentives for plaintiffs to settle under the shadow of the law, and for defendants to generate test cases, and investigate how the static and dynamic efficiency of the law is effected by these strategic actors.

This paper is organized as follows. Section 2 presents the basic legal framework. The third studies the static game and the fourth studies the dynamic (two-period) game with long lived plaintiffs. Section 5 considers two extensions and the final section concludes.

2 Model

We adapt the model in Parameswaran (2018). Consider a two-period game with three players: a firm, a regulator and a court. In each period $t \in \{1, 2\}$, a risk neutral, profit-maximizing firm must choose a quantity of output, q_t, to produce. The firm's gross (per-period) profit from production is $q_t - \frac{1}{2}q_t^2$, which is maximized at $q_{max} = 1$. Production creates a negative externality that harms a victim. The size of the harm is θq_t, where $\theta \in (0, 1)$ is the constant marginal harm. Accordingly, the socially efficient level of output is $q_{eff} = 1 - \theta < q_{max}$, which implies that unregulated firms will over-produce. Since firms will not produce the socially efficient output, there is scope for welfare-improving regulation.

[4] Hadfield (1991) makes a similar argument in an informal model, although her argument depends crucially on agent heterogeneity.

The regulator acts as an agent of the victims, on whose behalf it may seek compensatory damages. Thus, the regulator takes the role of the plaintiff, whilst the firm is the defendant. If θ were known, then the enforcement problem faced by a regulator is well-studied (see Becker, 1968). Instead, in this context, the regulator cannot always summarily sanction a firm because it will not always know with certainty whether the firm's output is inefficiently large or not. This uncertainty introduces the need for legal institutions (courts and the common law) that clarify the law for all parties by determining whether actions of the firm should be permitted or sanctioned. Thus, because of the uncertainty regarding θ, the court and regulator jointly enforce this regulation.

We model this uncertainty by assuming that, when the game begins (at $t = 1$), all players commonly believe that $\theta \sim U[l_1, u_1]$, where $0 \leq l_1 < u_1 \leq 1$. The uniformity assumption simplifies the analysis, since it ensures that the posterior distribution of beliefs after learning remains uniform. For similar technical reasons to Parameswaran (2018), we assume that $u_1 \leq \frac{2+l_1}{3}$.

Suppose, at period t, the agents believe that $\theta \sim U[l_t, u_t]$. Then all players know that an output $q_t < 1 - u_t$ will definitely not exceed the socially efficient level and that an output $q_t > 1 - l_t$ definitely will. The players do not know whether quantities produced in the region $q_t \in (1 - u_t, 1 - l_t)$ are inefficiently large or not.

An efficiency-minded court seeks to implement a negligence-type rule that holds the firm liable only when it has produced more than the true socially efficient quantity q_{eff}. Given the uncertainty about θ, the court actually implements an incomplete negligence rule; it holds a firm liable whenever it is sure that it has over-produced (i.e. if $q_t \geq 1 - l_t$) and not liable whenever it is sure that it has not over-produced (i.e. if $q_t \leq 1 - u_t$). When the court is uncertain about whether the firm has over-produced or not, it conducts an evidentiary trial to determine how the case should be decided.[5]

[5]We follow Niblett (2013) in assuming that the court always implements *narrow* rules — i.e. it makes

In the baseline analysis, we assume that learning at trial is partial: the agents merely learn whether the firm overproduced (i.e. if $q_t > 1-\theta$) or not. Hence, the learning technology truncates the support of the belief distribution. If the players learn that q_t was inefficiently large (i.e. $\theta > 1-q_t$), then the posterior beliefs will be $\theta \sim U[1-q_t, u_t]$. By contrast, if the players learn that q_t was not inefficiently large (i.e. $\theta < 1-q_t$), then the posterior beliefs will be $\theta \sim U[l_t, 1-q_t]$. These posterior beliefs will also be the players' prior beliefs at the beginning of the subsequent period $t+1$. As an extension, we consider the stronger case where learning at trial is complete, and the true θ is revealed to all players.

Let $I(q_t, l_t, u_t)$ denote the damages that the firm expects to pay if it produces output q_t and the case goes to trial. If $q_t \leq 1-u_t$, then the court summarily dismisses the case and no damages are paid. If $q_t \geq 1-l_t$, then the firm will pay full expectation damages $E[\theta]q_t = \frac{1}{2}(u_t+l_t)q_t$. Finally, if $q_t \in (1-u_t, 1-l_t)$, the firm will pay damages provided that $q_t > 1-\theta$. In expectation it pays: $\int_{l_t}^{u_t} \theta q_t \mathbf{1}[q_t > 1-\theta]\frac{1}{u_t-l_t}d\theta$. We have:

$$I(q_t, l_t, u_t) = \begin{cases} 0 & \text{if } q_t \leq 1-u_t \\ \frac{u_t^2-(1-q_t)^2}{2(u_t-l_t)} \cdot q_t & \text{if } q_t \in (1-u_t, 1-l_t) \\ \frac{1}{2}(u_t+l_t)q_t & \text{if } q_t \geq 1-l_t \end{cases}$$

Since our focus is on the strategic incentives to litigate (or not), we abstract from the assumption that the plaintiff/regulator and defendant/firm face trial costs. **Giri, I'm wondering whether it makes sense to define static profits and $J(.)$ here.**

The timing of each stage game is as follows: First the firm produces output q_t. The regulator may then initiate litigation. If so, the litigants engage in a bargaining process (described

no commitment about how it will decide cases when the correct disposition is unknown. This is in contrast to Baker and Mezzetti (2012) and Parameswaran (2018), where the court optimally implements broad rules, by summarily disposing of certain cases, for which the correct disposition is not known, but about which it has strong beliefs. Since our focus in this behavior is on the behavior of litigants and the strategic role of settlement, we abstract from considerations of optimal behavior by the court itself.

below) which may result in an out of court settlement s_t. In the case of settlement, the firm pays s_t to the regulator, and in doing so, both parties avoid trial and there is no learning. If there is no settlement, then the regulator must decide whether to proceed to trial or to drop the case. If the case proceeds to trial, then learning occurs as described above, and the defendant becomes liable for compensatory damages if $q_t > q_{eff} = 1 - \theta$.

In the baseline analysis, we consider a simple bargaining framework, under which the plaintiff makes a take-it-or-leave-it offer to the defendant, wherein the defendant pays the plaintiff $s \geq 0$ to settle out of court. As an extension, in Section 5.1, we consider a more general bargaining framework in which s is determined by asymmetric Nash Bargaining between the plaintiff and defendant, where the disagreement payoff is the subgame perfect payoff when settlement fails. parameter $\phi \in [0, 1]$ captures the bargaining strength of the plaintiff. The Nash bargaining approach includes, as special cases, the well studied scenarios where either the plaintiff ($\phi = 1$) or the defendant ($\phi = 0$) can make a take-it-or-leave-it offer. Thus the Nash Bargaining framework embeds our simplified baseline model as a special case. When $\phi \in (0, 1)$, Imai and Salonen (2000) and Parameswaran et al. (2021) show that the Nash Bargaining approach coincides with the limit case of bargaining between the players à la Baron and Ferejohn (1989) as players can make arbitrarily rapid counter-proposals, where ϕ is the plaintiff's recognition probability.

The plaintiff and defendant discount the future according to discount factors δ_P and δ_D, respectively, where $\delta_P, \delta_D \in [0, 1]$. If $\delta_i = 0$ for agent $i \in \{P, D\}$, then we say that agent i is myopic. The larger is δ_i, the more far-sighted the agent becomes, and the more weight they put on future outcomes.

A strategy for the firm is a function $q_t \in [0, 1]$ that determines a quantity to be produced in each period, as well as a decision $a_t(q_t, s_t) \in \{0, 1\}$ to accept or an offer s_t given q_t. A strategy for the plaintiff is a take-it-or-leave-it offer $s_t(q_t) \in \mathbf{R}$ to settle out of court, as well as a decision $b_t(q_t) \in \{0, 1\}$ about whether to proceed to trial or to drop the case in the event that the offer is rejected. All these functions will generically be functions of beliefs (l_t, u_t) and the discount factors (δ_P, δ_D). A quadruple $\{q_t, s_t, a_t, b_t\}$ is a Bayesian Perfect Equilibrium if, for each t:

1. q_t maximizes the firm's expected discounted stream of profits, taking as given the future behavior of the plaintiff, the anticipated bargaining outcome, and the evolution of the legal rule.

2. s_t maximizes the plaintiff's payoff, taking as given the defendant's acceptance strategy, the plaintiff's litigation choice, and the evolution of the legal rule.

3. a_t maximizes the firm's expected discounted stream of profits, taking as given the settlement offer, and the evolution of the legal rule.

4. b_t minimizes the plaintiff's expected discounted stream of uncompensated harms from the externality, anticipating the evolution of the legal rule.

3 Static Benchmark

We begin by characterizing equilibrium in a static version of the game, where there are no dynamic considerations. This will also be the equilibrium play in the second period (after which the game ends) and in the first period of the game provided that $\delta_P = 0 = \delta_D$. Without confusion, we omit time subscripts.

We solve the game by backward induction. First, consider the history where the firm has produced q and the plaintiff's settlement offer s has been rejected by the firm. The case will proceed to trial whenever the expected value to the plaintiff $I(q, l, u)$ from doing so is positive. This will be the case whenever $q > 1 - u$. If $q \leq 1 - u$, then the plaintiff is indifferent between litigating and not — and so by assumption will not — though her payoff would be the same if she did.

Next, consider the firm's decision to accept a settlement offer or not. The firm's expected payoff from rejecting the offer is $-I(q,l,u)$ (which will be negative if $q > 1-u$, and zero otherwise). The firm's payoff from accepting the offer is $-s$. Thus, the firm will accept a settlement offer if and only if $s \leq I(q,l,u)$.

Now, consider the plaintiff's optimal settlement offer. The plaintiff's expected payoff from making an offer that is rejected is $I(q,l,u)$ (which will be positive if $q > 1-u$ and zero otherwise). Thus an optimal settlement offer for the plaintiff must satisfy $s \geq I(q,l,u)$. Since the firm will only accept offers with $s \leq I(q,l,u)$, the equilibrium offer must be $s = I(q,l,u)$. Intuitively, since litigation is costless, there is no positive surplus from settlement. Instead settlement represents a zero-sum game, and any gains (over disagreement) to one player must result in a loss (over disagreement) to the other. The only possible settlement must coincide with the disagreement payoff. It will simply be the expected penalty from going to trial. Given this offer, both the plaintiff and defendant are indifferent between having the offer accepted and rejected, and by our assumptions, offers are accepted whenever the agents are indifferent.

Anticipating this settlement dynamic, the firm will choose its quantity to maximize:

$$\max_q \pi(q,l,u) = q - \frac{1}{2}q^2 - I(q,l,u)$$

Notice that this is identical to the problem that the firm would face in a world where the possibility of settlement was absent. In the static game, settlements do not distort the firm's choice.

Let $q_A(l,u) = 1 - \frac{1-(u-l)+\sqrt{(1-(u-l))^2+3u^2}}{3}$. This is the solution to the firm's first order condition when the firm produces in the region $q \in (1-u, 1-l)$. It is easily verified that

$q_A < 1 - \frac{u+l}{2} = q_E$, where q_E is the *ex ante* socially efficient output. Uncertainty about the law over-deters the firm, causing it to produce below the socially efficient level.

The firm's equilibrium output choice is given by:

Lemma 1. *The firm's optimal static choice $q^*(l,u)$ is given by:*

$$q^*(l,u) = \begin{cases} q_A(l,u) & u - l > 1 - u \\ 1 - u & u - l \leq 1 - u \end{cases}$$

Lemma 1 is a special case of Proposition 1 in Parameswaran (2018). For a detailed explanation of the properties of the proposition, we refer the interested reader to that paper. Here we simply describe the main feature salient to this paper.

The firm's decision amounts to choosing between two output levels, which we refer to as the 'safe' and 'risky' options. The safe option is the highest output that will definitely not attract litigation, $q = 1 - u$. The risky option is the output that maximizes profit (including expected penalties) within the ambiguous region (i.e. $q = q_A$). It is never optimal to choose an output for which the firm will definitely be penalized. Lemma 1 states that the firm will choose the 'risky option' when there is a lot of uncertainty about the efficient quantity, and will choose the 'safe option' otherwise. Choosing the risky option has the benefit of earning the firm larger pre-penalty profits, but this comes with the risk of incurring a penalty. If the safe quantity is too low, then the pre-penalty profits forgone from choosing the safe option will be large, and this incentivizes the firm to choose the risky option. By contrast, if the safe quantity is sufficiently large, the pre-penalty profits forgone will be low relative to the expected penalty from the risk option, and so the safe option is preferred.

How large the safe output needs for it to be chosen depends the extent of uncertainty about θ. Indeed, as uncertainty increases (i.e. $u - l$ becomes larger) the safe output must also be larger in order to be chosen in preference to the risky output. The intuition is that, as $u - l$

becomes larger, a marginal increase in q increases the probability of having over-produced by less. Moreover, when the degree of uncertainty is sufficiently small ($u - l \leq 1 - u$), the firm is guaranteed to choose the safe option.

Before moving the the dynamic analysis, we introduce some notation that will prove useful. Let $J(l, u)$ denote the expected residual harm suffered by the victim when the firm produces the optimal static output $q^*(l, u)$ after accounting for expected damages awarded at trial. We have:

$$J(l, u) = q^*(l, u) \cdot \frac{u + l}{2} - I(q^*(l, u), l, u)$$

Similarly, let $\Pi(l, u) = \pi(q^*(l, u), l, u)$ denote the firm's expected static profit (inclusive of expected penalties at trial) when it produces the optimal static output $q^*(l, u)$.

4 Dynamic Analysis

We now begin the dynamic analysis. Let δ_P, δ_D take generic values in $[0, 1]$. First, consider behavior in the second period. Since the game ends after the second period, equilibrium behavior will coincide with that in the static game, which we described in the previous section.

4.1 Credible Trials and Learning

Now, move to the first period, and consider a history in which the firm has produced q_1 and has rejected the plaintiff's offer s. The plaintiff must decide whether to proceed to trial or drop the litigation. If either $q_1 \leq 1 - u_1$ or $q_1 \geq 1 - l_1$, then there will be no learning at trial, so the second period outcomes will be independent of whether the case proceeds to trial or not. Furthermore, if $q_1 \leq 1 - u_1$, then the victim recovers no damages from going to trial. In equilibrium she will drop the litigation. By contrast, if $q_1 \geq 1 - l_1$, then the victim receives full expectation damages from going to trial without affecting second period outcomes; she will definitely proceed to trial.

The more interesting case arises when $q_1 \in (1-u, 1-l)$. Here, the victim faces a trade-off. On the one hand, proceeding to trial generates positive expected damages. On the other hand, as we verify below, learning at trial harms the victim in expectation, since it allows the firm to more finely target its second period output at a level that makes it less likely to be penalized, thus leaving the victim to bear a larger share of second-period harms, in expectation.

In deciding whether to proceed to trial, the plaintiff must trade-off the current expected gains against the future expected losses. She will go to trial provided that:

$$I(q_1, l_1, u_1) - \delta_P \left[\frac{u_1 - (1-q_1)}{u_1 - l_1} J(1-q_1, u_1) + \frac{(1-q_1) - l_1}{u_1 - l_1} J(l_1, 1-q_1) \right] > -\delta_P J(l_1, u_1)$$
$$I(q_1, l_1, u_1) > \delta_P \mathcal{A}_P(q_1, l_1, u_1)$$

where $\mathcal{A}_P(q_1, l_1, u_1) = \frac{u_1-(1-q_1)}{u_1-l_1} J(1-q_1, u_1) + \frac{(1-q_1)-l_1}{u_1-l_1} J(l_1, 1-q_1) - J(l_1, u_1)$.

$\mathcal{A}_P$ denotes the net expected second period loss that the plaintiff suffers from having a case go to trial rather than not. The first two terms together represent the expected residual second period harms born by the victim when there is learning (which depends on what is learned), while the third term represents the same without learning.

Analogously, denote by $\mathcal{A}_D(q_1, l_1, u_1)$ the net expected gain to the defendant from going to trial rather than not. We have:

$$\mathcal{A}_D(q_1, l_1, u_1) = \frac{u_1 - (1-q_1)}{u_1 - l_1} \cdot \Pi(1-q_1, u_1) + \frac{(1-q_1) - l_1}{u_1 - l_1} \cdot \Pi(l_1, 1-q_1) - \Pi(l_1, u_1)$$

Lemma 2. *Learning is beneficial to the firm and costly to the victim. Formally,* $\mathcal{A}_P(q, l, u) \geq 0$ *and* $\mathcal{A}_D(q, l, u) \geq 0$ *for all* $q \in [1-u, 1-l]$.

Lemma establishes that learning is beneficial to the firm and costly to the victim (in expectation). It allows the firm to more finely target its second period output, thereby enabling it earn larger profits whilst facing a lower risk of penalty. Precisely because it enables the firm to more readily avoid sanction, it transfers more of the burden of harms onto the victim, thus making her worse off.

Return to the victim's decision about whether to proceed to trial in the event that pre-trial bargaining fails. The victim must trade-off the expected benefit of receiving damages in the current period against the expected future cost that stems from learning. Naturally, the trade-off will depend on the size of the discount factor. In fact, as we establish in the following Lemma, for any $\delta_P \in [0, 1]$ the trade-off always resolves in favor of going to trial, whenever the plaintiff expects positive damages at trial (i.e. $q_1 > 1 - u_1$).

Lemma 3. *For any $\delta_P \in [0, 1]$, if $q_1 > 1 - u_1$, then the plaintiff strictly prefers to go to trial than to drop litigation.*

Thus, if settlement fails, the case will proceed to trial whenever doing so gives the plaintiff positive damages in expectation. In the dynamic game, going to trial is individually rational for the plaintiff whenever it would be in the static game — dynamic considerations do not distort the plaintiff's incentive to litigate.

4.2 Settlement

Now, move to the settlement phase, anticipating that the dispute will proceed to trial whenever bargaining fails (assuming $q_1 > 1-u_1$). As discussed in the previous section, if $q_1 \leq 1-u_1$ or $q \geq 1 - l_1$, there is no possibility of learning, and so there are no dynamic linkages across the periods. The incentives for all players reflect the incentives in the static game, and thus the equilibrium settlement dynamic will match that in the static game.

The more interesting case arises when $q_1 \in (1-u_1, 1-l_1)$, since in this scenario, learning will occur if the case proceeds to trial. As we noted, this learning benefits the firm and harms the plaintiff. The firm may nevertheless be willing to accept a settlement offer s provided that what it saves in expected first period damages exceeds what it loses by preventing learning:

$$\begin{aligned}-s+\delta_D \Pi(l_1,u_1) &\geq -I(q_1,l_1,u_1)+\delta_D\left[\frac{u_1-(1-q_1)}{u_1-l_1}\cdot\Pi(1-q_1,u_1)+\frac{(1-q_1)-l_1}{u_1-l_1}\cdot\Pi(l_1,1-q_1)\right]\\ -s &\geq -I(q_1,l_1,u_1)+\delta_D\mathcal{A}_D(q_1,l_1,u_1)\end{aligned}$$

The terms on the right-hand-side denote the firm's expected (net) payoff from going to trial, which is the sum of the current expected penalty $I(q_1,l_1,u_1)$ and the (discounted) future gains to profit that stem from learning $\delta_D\mathcal{A}_D(q_1,l_1,u_1)$.

The plaintiff/victim will accept a settlement s rather than going to trial if:

$$\begin{aligned}s-\delta_P J(l_1,u_1) &\geq I(q_1,l_1,u_1)-\delta_P\left[\frac{u_1-(1-q_1)}{u_1-l_1}\cdot J(1-q_1,u_1)+\frac{(1-q_1)-l_1}{u_1-l_1}\cdot J(l_1,1-q_1)\right]\\ s &\geq I(q_1,l_1,u_1)-\delta_P\mathcal{A}_P(q_1,l_1,u_1)\end{aligned}$$

The second line precisely expresses the trade-off of going to trial discussed above. The current expected benefit of going to trial is $I(q_1,l_1,u_1)$, while the future (discounted) cost is $\delta_P\mathcal{A}_P(q_1,l_1,u_1)$. The net benefit of going to trial is the sum of these, and the plaintiff will accept any settlement s that is strictly larger.

Combining these, we find that the surplus from settling is: $\delta_P\mathcal{A}_P-\delta_D\mathcal{A}_D$. The first term in this difference represents the gain to the plaintiff from preventing learning, while the second term represents the loss to the defendant from not learning. The surplus from settling will be positive provided that:

$$\frac{\delta_P}{\delta_D}\geq\frac{\mathcal{A}_D(q_1,l_1,u_1)}{\mathcal{A}_P(q_1,l_1,u_1)} \tag{1}$$

We refer to equation (1) as the *settlement condition*. The settlement condition requires that

δ_P is large relative to δ_D — the plaintiff must value the future by relatively more than the defendant, which ensures that the future benefits (to the plaintiff) of preventing learning outweigh the costs (to the defendant).

If this condition is satisfied, then there exists a settlement $s \geq 0$ that makes both agents better off than going to trial. If so, we should expect settlement along the equilibrium. Given our (simplifying) assumption that the plaintiff makes a take-it-or-leave-it offer to the defendant, we should expect the plaintiff to extract the entire surplus from settlement, leaving the defendant indifferent between settling or not. Thus, whenever settlement arises along the equilibrium path, we should expected a settlement amount:

$$s(q_1, l_1, u_1) = I(q_1, l_1, u_1) - \delta_D \mathcal{A}_D(q_1, l_1, u_1)$$

Notice that $s(q_1, l_1, u_1) \leq I(q_1, l_1, u_1)$, and so, in equilibrium, the parties will settle for less than the expected damages that would be awarded at trial in the first period. Intuitively, since the plaintiff is harmed by learning, she will be willing to accept a lower settlement to prevent the case from going to trial and learning from happening. Conversely, since the defendant benefits from learning, she will be unwilling to acquiesce to a settlement, unless it is much cheaper than the expected penalty that she would pay if she went to trial.

Finally, if the settlement condition (1) is not satisfied, then there is no settlement offer that can satisfy both agents; at least one of the parties will be worse off from the settlement. The agents will proceed to trial along the equilibrium path.

4.3 Firm's Quantity

Finally, consider the firm's choice of first period quantity, anticipating the settlement dynamics that follow. Recall, if there is a settlement, it leaves the defendant just as well as it would've been without a settlement. Thus, the firm's decision making is tantamount to maximizing its expected lifetime profit absent settlement. We have:

$$\max_{q_1} \pi(q_1, l_1, u_1) + \delta_D \left[\Pi(l_1, u_1) + \mathcal{A}_D(q_1, l_1, u_1)\right]$$

which is the sum of expected first period profits (inclusive of penalties) and the expected second profits which depend on the firm's first period learning.

By the first order conditions, we know that $\frac{\partial \pi(q^*(l_1,u_1),l_1,u_1)}{\partial q} \leq 0$.[6] Hence, from the perspective of first period expected profits, there is no incentive for the firm to choose a quantity above the static benchmark $q^*(l_1, u_1)$. By contrast, as we establish in the following lemma, from the perspective of second period expected profits, the learning motive provides a strict incentive to produce beyond the static benchmark.

Lemma 4. *The following are true:*

$$\frac{\partial \mathcal{A}_D(q^*(l,u), l, u)}{\partial q} > 0 \quad \textit{and} \quad \frac{\partial \mathcal{A}_D(q_E(l,u), l, u)}{\partial q} < 0$$

Lemma 4 implies that the second period component of the firm's lifetime profits is maximized by choosing a riskier output than the optimal static choice $q^*(l, u)$. (This follows since $\frac{\partial \mathcal{A}_D(q^*(l,u),l,u)}{\partial q} > 0$.) However this riskier output is not the one that maximizes learning in the sense of minimizing the expected second period variance in beliefs about θ; instead learning is maximized when $q_1 = q_E$ — i.e. at the socially efficient output. Even when it is focused purely on maximizing expected second period profits, the firm under-produces relative to the efficient benchmark ($q_1 < q_E$). Doing so makes it more likely that it will receive 'good news' at trial (i.e. that $\theta < 1 - q_1$) and thus be able to produce a larger output in the second period, than that it will receive 'bad news' (i.e. that $\theta > 1 - q$).

[6]In fact, the condition holds with equality when $u_1 - l_1 \geq 1 - u_1$, which implies an interior solution $q^*(l_1, u_1) = q_A(l_1, u_1)$. However, when $u_1 - l_1 < 1 - u_1$, there is a corner solution $q^*(l_1, u_1) = 1 - u_1$ at which the first derivative may be negative.

Of course, the firm must optimally trade-off the first period cost from choosing a risky output against the second period gain. The firm's optimal first period quantity satisfies:

$$\frac{\partial \pi(q_1, l_1, u_1)}{\partial q_1} + \delta_D \frac{\partial \mathcal{A}_D(q_1, l_1, u_1)}{\partial q} \leq 0 \tag{2}$$

Furthermore, the condition will hold with equality, with the possible exception of cases where the static best choice is a corner solution. (We will investigate this case further, below.)

Let $\hat{q}_1(l_1, u_1, \delta_D)$ denote the firm's optimal first period output. At the optimal first period output $\hat{q}_1$, we have $\frac{\partial \pi}{\partial q} < 0$ and $\frac{\partial \mathcal{A}_D}{\partial q} > 0$, so that the defendant produces a higher output than a purely short-term oriented firm would, but a lower output than a purely learning-oriented firm would. Naturally, as δ_D increases, the learning motive becomes more salient, and so more patient firms will tend to produce larger outputs. This gives the following result:

Proposition 1. *The firm's equilibrium first period output $\hat{q}_1(l_1, u_1, \delta)$ satisfies $\hat{q}_1 \in [q^*(l_1, u_1), q_E(l_1, u_1))$. Additionally, $\hat{q}_1(\delta_D) = q^*$ when $\delta_D \leq \overline{\delta}_D(l_1, u_1) = \max\left\{0, \frac{1-u_1}{u_1-l_1} - 1\right\}$, and $\hat{q}_1(\delta_D)$ is strictly increasing in δ_D for $\delta_D > \overline{\delta}_D$.*

Similar to Lemma 1, the properties of the firm's first period output choice depends on the extent of uncertainty. If uncertainty is relatively large (i.e. if $u_1 - l_1 > 1 - u_1$), then the optimal output is an interior solution to the first order conditions and is strictly monotone in δ_D. Indeed, for any δ_D, the firm will produce a 'risky' output that will leave it potentially liable at trial, and this risky quantity rises as the firm becomes more patient. By contrast, when uncertainty is relatively low (i.e. if $u_1 - l_1 \leq 1 - u_1$), then a sufficiently impatient firm chooses the 'safe' output, making it immune to liability. Nevertheless, once it becomes sufficiently patient (i.e. if $\delta_D > \frac{1-u_1}{u_1-l_1} - 1$), it will take risky actions that open it to liability.

Proposition 1 shows how the firm's incentive to learn affects its first period choice. Relative to the static baseline $q^*(l, u)$, dynamic incentives cause the firm to produce a (weakly) larger output. In doing so, the firm optimally trades-off the cost of higher expected penalties in the first period, against the benefit of higher second period profits which arise because the firm can make choices with greater certainty about likely penalties. Importantly, the Proposition shows that the 'distortion' (relative to the baseline) is in the direction of the *ex ante* efficient output level. This follows because the stage-game efficient output level is also the one that maximizes the benefit from learning. Finally, the size of the 'distortion' is increasing in the firm's degree of patience; more long run oriented firms will be more inclined to choose outputs that incur a penalty in the first period, but which generate higher second period benefits from learning.

4.4 Equilibrium

We are now ready to characterize the equilibrium of the game. There will be two cases to consider, depending on whether the extent of uncertainty is small or large (which implicates whether the optimal solution is possibly at the the corner or interior).

Proposition 2.A. *Suppose the extent of uncertainty is small (i.e. $u_1 - l_1 \leq 1 - u_1$) and define $\tilde{\delta}_P(\delta_D) = \frac{u_1+1-\hat{q}_1(\delta_D)}{l_1+1-\hat{q}_1(\delta_D)}\delta_D$. Then:*

1. *If $\delta_D \leq \overline{\delta}_D(l_1, u_1)$, then the firm's first period quantity will be $\hat{q}_1(l_1, u_1, \delta_D) = q^*(l_1, u_1) = 1 - u_1$. The plaintiff will not initiate litigation along the equilibrium path.*

2. *If $\delta_D > \overline{\delta}_D(l_1, u_1)$, then the firm's first period quantity will be $\hat{q}_1(l_1, u_1, \delta_D) > q^*(l_1, u_1) = 1 - u_1$. There will be litigation along the equilibrium path. There will be settlement if $\delta_P \geq \tilde{\delta}_P(\delta_D)$; otherwise the parties will proceed to trial. Additionally, the locus $\tilde{\delta}_P(\delta_D)$ is strictly increasing in δ_D.*

Proposition 2.A divides the parameter space into three regions. As we showed in Proposition 1, when the defendant is sufficiently impatient (i.e. $\delta_D \leq \overline{\delta}_D$), then the firm chooses the safe level of output and there is no litigation or learning. If instead the defendant if patient enough

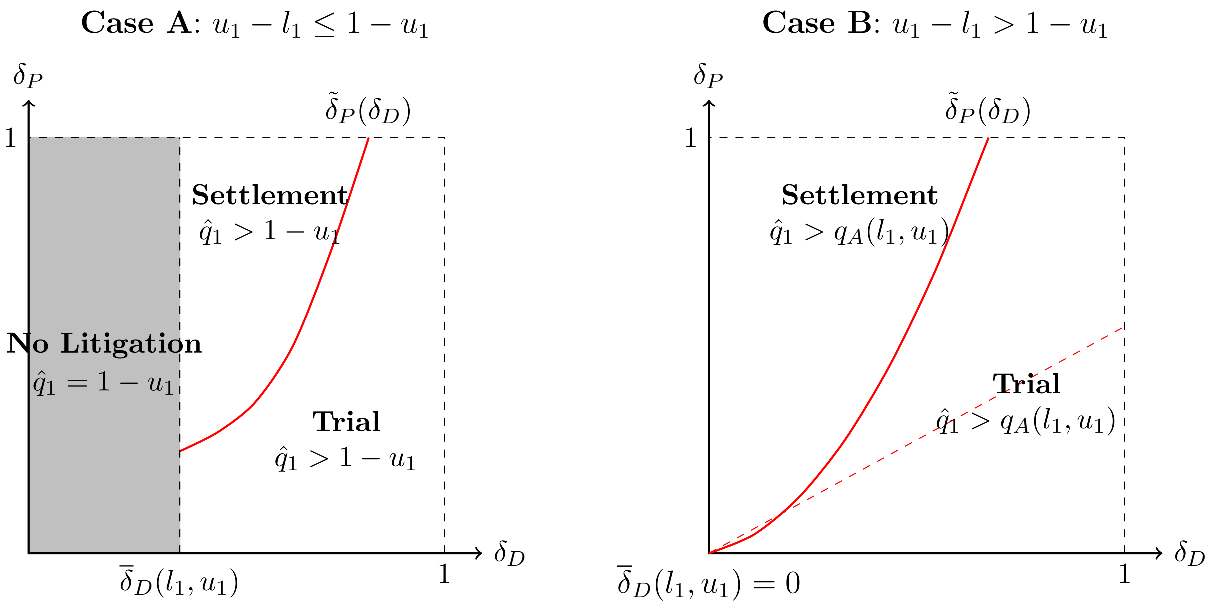

Figure 1: The left and right panels show the litigation dynamics when $u_1 - l_1 \leq 1 - u_1$ and $u_1 - l_1 > 1 - u_1$ (respectively). The key difference is that, in the former case, there may be no litigation at all. The dashed red line in the right panel represents the counterfactual settlement condition when the firm produces the statically optimal output q_A, but the settlement decision is dynamically optimal.

(i.e. $\delta_D > \overline{\delta}_D$), then the firm chooses a quantity above the safe level which makes litigation credible, since the plaintiff can now expect positive expectation damages. Consistent with the settlement condition, when the plaintiff is relatively more patient than the defendant, the plaintiff's incentive to settle dominates — she will accept a much lower settlement offer than would be optimal in the static game, which makes settlement optimal for the defendant (despite forgoing the benefits of learning). By contrast, when the defendant is relatively more patient, the defendant's incentive to learn dominates. There is no (incentive compatible) settlement offer that the plaintiff can make that the defendant would accept; the case will go to trial. These three regions are represented in Panel A of Figure 1.

Proposition 2.B. *Suppose $u_1 - l_1 > 1 - u_1$. The firm's first period output will be larger than in the static baseline: $\hat{q}_1(l_1, u_1, \delta_D) > q^*(l_1, u_1)$ (unless $\delta_D = 0$). Along the equilibrium path, there will be settlement if the plaintiff is sufficiently patient relative to the defendant — i.e. if $\delta_P \geq \tilde{\delta}_P(\delta_D)$, where $\tilde{\delta}_P(\delta_D) = \frac{\mathcal{A}_D(\tilde{q}_1(l_1,u_1,\delta_D))}{\mathcal{A}_P(\tilde{q}_1(l_1,u_1,\delta_D))}\delta_D$.*

Proposition 2.B is analogous to Proposition 2.A, except that the firm will always produce a risky output and so the plaintiff will always initiate litigation. The difference arises because when the degree of uncertainty is relatively high (i.e. $u_1 - l_1 > 1 - u_1$), even in the static game, the defendant will choose the risky output, which makes litigation incentive compatible. By contrast, when the degree of uncertainty is low (i.e. $u_1 - l_1 < 1 - u_1$), the defendant will make the safe choice in the static game and indeed, even in the dynamic game unless it is sufficiently forward looking.

Several features of the equilibrium are worth noting. First, the model predicts both settlements and trials along the equilibrium path, depending on the relative degrees of patience of the plaintiff and defendant. Results are stark in two focal extremes: if the plaintiff is long-run oriented whilst the defendant is short-run oriented, then settlement is likely along the equilibrium path. This may well capture the dynamic between regulators (who take the long-run view) interacting with a sequence of short-run oriented firms. If so, the dynamic incentives will cause the parties to settle out of court, leaving the law unsettled and hampering legal evolution. By contrast, if the roles are reversed so that the regulator is short-run oriented whilst the defendant is forward looking, then cases will tend to go to trial. Legal evolution requires that the more long-run oriented party also be the one most invested in having the law clarified.

Second, dynamic considerations will generally cause the firm to produce a riskier output than it would if it were purely present biased. Since the firm benefits from learning, it has a strong incentive to test the boundaries of the law, enabling it to make choices with greater certainty in the future if the case goes to trial. This dynamic is most clear in Case A where the extent of uncertainty of small, such that the firm would choose the safe output in the static model, but instead chooses a risky one in the dynamic model (provided that it is sufficiently patient).

Third, and somewhat more subtly, dynamic incentives for the firm result in an output choice that make settlement less likely (than would have been the case had the firm made the optimal static choice). To fix ideas, take Case B where the extent of uncertainty is relatively large, and where even in the static model, the firm would choose the risky output $q_A(l_1, u_1)$ and thus litigation would arise. Consider the counterfactual dynamic game in which the firm continued to produce q_A, but where the subsequent litigation and settlement choices were dynamically optimal for the parties (i.e. the costs and benefits of learning were appropriately accounted for). The dashed red line in the right panel of Figure 1 represents the settlement locus in this counterfactual. Notice that the dashed line lies below the true settlement locus that arises in equilibrium. Thus, for any δ_D, there will be a range of δ_P for which the parties would have settled in the counter-factual where the firm made the statically optimal choice, but instead go to trial in equilibrium because the firm's dynamically optimal choice was riskier.

Fourth, notice that the defendant's first period output choice depends only on her own degree of patience, and is independent of the plaintiff's. This followed from our assumption that the plaintiff could make a take-it-or-leave-it settlement offer to the defendant, which, in equilibrium, would leave the plaintiff indifferent between settling and going to trial, whenever settlement was equilibrium consistent. Thus, the firm's equilibrium payoff is identical to a counter-factual model where settlement was disallowed and litigation always proceeded to trial. (We discuss the implications of relaxing this assumption in Section 5 below.)

Before concluding this section, we explore some efficiency implications of the dynamic model. In doing so, we distinguish two dimensions of efficiency. The first, 'static efficiency', is an intra-period notion efficiency which measures how close the firm's first period output is to the efficient benchmark $q_E(l_1, u_1) = 1 - \frac{u_1+l_1}{2}$. The second, 'dynamic efficiency', is an inter-period notion of efficiency related to how much learning occurs in the first period (which in turn enables greater second period 'static' efficiency). We note that these two efficiency notions need not be in conflict: on both dimensions the efficient choice is for the firm to choice $q_E(l_1, u_1)$ in the first period.

Fix the plaintiff's degree of patience δ_P. Proposition 1 shows that the firm's first period output becomes more statically efficient as the defendant becomes more patient (i.e. as δ_D). The firm's incentive to learn causes it to better internalize the externality it imposes on the victim, thereby producing an output closer to (but not at) the efficient benchmark q_E. Propositions 2.A and 2.B show that as the firm becomes more patient, the 'likelihood' of the case proceeding to trial and thus resulting in legal evolution, increases.[7] This is the extensive margin of dynamic efficiency. Additionally, there is an intensive margin: as δ_D increases, the extent of learning increases (in the sense of reducing the variance in beliefs about θ by a larger amount).

Taken together, we have shown that equilibrium outcomes are most efficient (though never achieve the efficient benchmark) when the defendant is increasingly patient but the plaintiff is relatively impatient. By contrast, outcomes are least efficient when the defendant is very impatient and the plaintiff is relatively more patient.

5 Extensions

In this section, we investigate two extensions to our baseline model. First, we relax the simplifying assumption that the plaintiff makes a take-it-or-leave-it during settlement negotiations. Second, we investigate outcomes under an alternative learning technology in which θ is fully revealed if the case goes to trial.

[7] By 'likelihood' we mean the range of parameter values for which the outcome is equilibrium consistent. Since (δ_P, δ_D) is common knowledge, there is no uncertainty about whether cases are settled or go to trial along the equilibrium path. However, taking δ_P as a publicly observed draw from a distribution (conditional upon δ_D), the likelihood of the draw resulting in settlement decreases as δ_D increases.

5.1 Settlement Bargaining

Consider a more general approach to settlement negotiation in which the settlement s is determined by asymmetric Nash Bargaining. The disagreement payoff is the subgame perfect equilibrium payoff when settlement fails, which will be the expected payoff from going to trial when this is individually rational for the plaintiff. Let $\phi \in [0,1]$ denote the plaintiff's bargaining strength. If $\phi = 1$, then Nash Bargaining reduces to the baseline case where the plaintiff makes a take-it-or-leave-it offer. When $\phi = 0$, we have the opposite case of a defendant making take-it-or-leave-it offers. When $\phi \in (0,1)$ bargaining power is shared between the parties.

Changing the bargaining protocol will not affect the parties' incentives to proceed to trial if settlement fails. Thus, as in the baseline model, we know that litigation is rational for the plaintiff whenever $q_1 > 1 - u_1$. Moreover, whenever the plaintiff cannot credibly go to trial, it will not be able to extract settlements from the defendant. Thus, we focus on the case when litigation is credible.

The incentive to settle is also unchanged under the modified bargaining protocol. The plaintiff will accept any settlement offer $s \geq I(q_1, l_1, u_1) - \delta_P \mathcal{A}_P(q_1, l_1, u_1)$. Analogously, the defendant will accept a settlement if $s \leq I(q_1, l_1, u_1) - \delta_D \mathcal{A}_D(q_1, l_1, u_1)$. The settlement surplus is: $\delta_P \mathcal{A}_P(q_1, l_1, u_1) - \delta_D \mathcal{A}_D(q_1, l_1, u_1)$, which is identical to baseline. Settlement will arise along the equilibrium path whenever the settlement surplus is non-negative, i.e. whenever the settlement condition (1) is satisfied. Thus, conditional on the firm's output, changing the bargaining protocol does not affect whether the litigants settle or proceed to trial.

However, it will affect the terms of settlement $\hat{s}(q_1, l_1, u_1)$, which is determined by:

$$\begin{aligned}\hat{s} &= \arg\max_s [s - I(q_1, l_1, u_1) + \delta_P \mathcal{A}_P(q_1)]^{\phi}[-s + I(q_1, l_1, u_1) + \delta_D \mathcal{A}_D(q_1)]^{1-\phi} \\ &= I(q_1, l_1, u_1) - \delta_P \mathcal{A}_P(q_1) + \phi[\delta_P \mathcal{A}_P(q_1) - \delta_D \mathcal{A}_D(q_1)]\end{aligned}$$

The equilibrium settlement gives the plaintiff her disagreement payoff plus a fraction $\phi \in [0, 1]$ of the settlement surplus. The defendant likewise receives his disagreement payoff plus a fraction $1 - \phi$ of the settlement surplus. Evidently, by varying ϕ, we can induce any division of the surplus between the litigants.

Taking the firm's period 1 output as given, generalizing the bargaining protocol did not affect the litigation dynamics. However, modifying the bargaining protocol will generically affect the firm's output choice when $\phi < 1$. To see this, notice that the firm's period 1 decision now becomes:

$$\max_{q_1} \begin{cases} \pi(q_1, l_1, u_1) + \delta_D \mathcal{A}_D(q_1) & \text{if } \frac{\delta_P}{\delta_D} \leq \frac{\mathcal{A}_D(q_1)}{\mathcal{A}_P(q_1)} \\ \pi(q_1, l_1, u_1) + \delta_D \mathcal{A}_D(q_1) + (1 - \phi)\left[\delta_P \mathcal{A}_P(q_1) - \delta_D \mathcal{A}_D(q_1)\right] & \text{if } \frac{\delta_P}{\delta_D} > \frac{\mathcal{A}_D(q_1)}{\mathcal{A}_P(q_1)} \end{cases}$$

Since the firm now receives a positive fraction of the settlement surplus, it will no longer be indifferent between settling or proceeding to trial. This affects its output choice in two ways. First, in choosing q_1 it must anticipate whether its choice will induce settlement or not along the equilibrium path. By the settlement condition, the possibility of settlement depends on the relative degrees of patience of both the plaintiff and defendant. Thus, and in contrast to Proposition 1, the firm's optimal output choice will generically depend on both δ_P and δ_D.

Second, relative to optimal dynamic output characterized in Proposition 1, the firm may now have an incentive to distort its output choice in order to increase the settlement surplus, and thus extract more concessions from the plaintiff during settlement negotiations. The direction of this distortion is somewhat complicated to predict, since it will depend on the interplay between q_1, δ_P and δ_D. We stress, however, that this incentive to distort arises only when the firm anticipates settlement along the equilibrium path. When it expects to go to trial, it faces the same incentives, and will thus make the same choice as it would in the baseline model.

5.2 Learning

In some situations, the law is immediately and fully clarified when the case goes to trial. For example, in a recent case (*Yegiazaryan v. Smagin*) the U.S. Supreme Court ruled that foreign nationals or foreign parties can file a claim under the Racketeer Influenced and Corrupt Organizations Act (RICO). Whereas, up to this point it had been unclear whether this was legally permissible in a U.S. Court. Similarly, as regards to copyright law, in Fourth Estate Public Benefit Corp. v. Wall-Street.com, LLC,[1] the U.S. Supreme Court decisively resolved that "a copyright infringement suit must wait until the copyright is successfully registered by the United States Copyright Office."[8]. Within the framework of our model, this implies that the exact value of θ is immediately learned as soon as a case goes to trial (instead of the distribution of θ narrowing, as we have assumed to this point).

In this extension, we assume that information revealed at trial is perfectly informative of the underlying externality (i.e. θ is perfectly revealed), and this results in the law being clarified. The court continues to implement a negligence rule, so the firm pays a penalty θq_1 whenever $q_1 > 1 - \theta$. Assuming that the firm produces in the ambiguous region, $q_1 \in (1 - u_1, 1 - l_1)$, the expected penalty that the firm expects to be assessed at trial is $E[\theta \,|\, \theta > 1 - q_1] \cdot q_1 = I(q_1, l_1, u_1)$. Thus, the modified learning technology does not change the expected penalty schedule, and the static optimal policy for the firm is as in Lemma 1.

[8]See `https://supreme.justia.com/cases/federal/us/586/17-571/`

The modified learning technology does change the costs and benefits of learning to the litigants. After the law is clarified, the firm will be able to produce the true socially efficient quantity $q^{eff} = 1 - \theta$ with impunity in the second period. By contrast, the plaintiff will suffer the full expected harm from the firm's (efficient) production $\theta(1-\theta)$. Thus, the firm's expected second period profit (if it goes to trial, but prior to the evidence being revealed) is $\frac{1}{2} - \frac{1}{2}E[\theta^2]$. By contrast, the plaintiff's expected second period harm is $E[\theta(1-\theta)]$. Unlike the baseline model, the second period payoffs (following learning) are independent of the firm's first period quantity q_1.

With these modified payoffs from learning, the incentives for litigation change. The plaintiff will find it rational to go to trial provided that:

$$\begin{aligned} I(q_1, l_1, u_1) - \delta_P E[\theta(1-\theta)] &> -\delta_P J(l_1, u_1) \\ I(q_1, l_1, u_1) &> \delta_P \mathcal{B}_P(l_1, u_1) \end{aligned}$$

where $\mathcal{B}_P(l_1, u_1) = E[\theta(1-\theta)] - J(l_1, u_1)$ denotes the net expected second period loss that the plaintiff suffers from having the case go to trial rather than not. Similarly, let $\mathcal{B}_D(l_1, u_1) = \frac{1}{2} - \frac{1}{2}E[\theta^2] - \Pi(l_1, u_1)$ denote the net expected second period gain to the defendant from having the law clarified.

The above inequality implies the following result:

Lemma 5. *There exists a threshold quantity (the de facto safe output) $\hat{\lambda}(\delta_P, l_1, u_1)$, characterized by: $I(\hat{\lambda}(\delta_P, l_1, u_1), l_1, u_1) = \delta_P \mathcal{B}_P(l_1, u_1)$, such that the plaintiff will credibly take the defendant to trial iff $q_1 > \hat{\lambda}(\delta_P, l_1, u_1)$. Moreover, $\hat{\lambda}$ is increasing in δ_P, $\hat{\lambda}(0, l_1, u_1) = 1 - u_1$ and $\hat{\lambda}(1, l_1, u_1) > q^*(l_1, u_1)$.*

Lemma 5 identifies an important distinction between the gradual learning rule of section 3 and the complete learning technology described here. Whereas in the baseline, litigation was always incentive compatible for the plaintiff, with complete learning, the plaintiff will only litigate if the firm's first period choice (and thus the expected damages from going to trial) are sufficiently large. $\hat{\lambda}(\delta_P, l_1, u_1)$ defines the threshold quantity, above which the plaintiff will litigate. Moreover, unless $\delta_P = 0$, $\hat{\lambda} > 1 - u_1$, and so there are a range of cases — where $q_1 \in (1 - u_1, \hat{\lambda}(\delta_P)]$ — that would be litigated in the static game, but not in the dynamic one. Over this range of cases, though the firm could potentially be held liable if taken to court, the plaintiff will not initiate litigation. $\hat{\lambda}$ becomes the *de facto* safe output for the firm, and the firm would never produce a quantity below this.

Importantly, the *de facto* safe output is increasing in the plaintiff's degree of patience. Intuitively, as the future harms from having the law clarified become more salient to the plaintiff, the upfront expected damages that she stands to recover must be commensurately higher to incentivize her to go to trial. This requires the firm to choose a higher quantity that is associated with larger expected damages.

The settlement decision with the modified learning technology mirrors that in the baseline model. The plaintiff will accept a settlement s whenever $s \geq I(q_1, l_1, u_1) - \delta_P \mathcal{B}_P(l_1, u_1)$. Similarly, the defendant will accept a settlement whenever $-s \geq -I(q_1, l_1, u_1) + \delta_D \mathcal{B}_D(l_1, u_1)$. The settlement surplus is $\delta_P \mathcal{B}_P(l_1, u_1) - \delta_D \mathcal{B}_D(l_1, u_1)$, and settlement will occur along the equilibrium path whenever the settlement surplus is non-negative — i.e. if $\frac{\delta_P}{\delta_D} \geq \frac{\mathcal{B}_D(l_1, u_1)}{\mathcal{B}_P(l_1, u_1)}$. This is straightforwardly analogous to the settlement condition (1) in the baseline analysis. Notice that the settlement condition is unaffected by the firm's period 1 quantity.

Next, consider the firm's first period decision. Notice that the firm's first period choice affects its second period profit only insofar as it affects whether there is litigation or not. Thus, if the optimal static choice is consistent with litigation by the plaintiff, then this choice must be globally optimal for the firm. By contrast, if the optimal static output does not attract litigation, then the firm will produce at least the *de facto* safe output, since it can so with effective immunity, and potentially slightly more if it seeks to guarantee that litigation arises along the equilibrium path.

For simplicity, return to our baseline assumption under which the plaintiff has all the bargaining power and can make a take-it-or-leave-it offer. Let $\underline{\delta}_P$ be such that $\hat{\lambda}(\delta_P) \geq q^*(l_1, u_1)$ whenever $\delta_P \geq \underline{\delta}_P$. With the above insights, we are ready to characterize the equilibrium. Suppose $u_1 - l_1 > 1 - u_1$ so that there is litigation in the baseline model.

Proposition 3. *There exist thresholds* $\tilde{\delta}_D(\delta_P) = \frac{\mathcal{B}_P}{\mathcal{B}_D}\delta_P - \frac{\max\{0, \pi(q^*(l_1,u_1),l_1,u_1) - \pi(\hat{\lambda}(\delta_P),l_1,u_1)\}}{\mathcal{B}_D(l_1,u_1)}$ *and* $\tilde{\delta}_P(\delta_D) = \frac{\mathcal{B}_D}{\mathcal{B}_P}\delta_D$ *such that, along the equilibrium path:*

1. *if* $\delta_D < \tilde{\delta}_D(\delta_P)$*, then the firm will produce the de facto safe output* $\hat{\lambda}(\delta_P)$*. There will be no litigation despite the expected damages at trial being positive. We say there is regulatory capture.*

2. *if* $\delta_D \geq \tilde{\delta}_D(\delta_P)$*, the firm will produce a risk output. Moreover:*

   - *If* $\delta_P > \underline{\delta}_P$*, the firm will produce just slightly more than the de facto safe output and take the case to trial, in effect bringing a test case.*

   - *If* $\delta_P \leq \underline{\delta}_P$*, the firm will produce the optimal stage game output output* $q^*(l_1, u_1)$*. The parties will settle if* $\delta_P > \tilde{\delta}_P(\delta_D)$ *and the case will go to trial otherwise.*

Proposition 3 is summarized in Figure 2 below. A comparison of Figures 1 and 2 allow us to identify the similarities and differences between the two learning technologies. In both cases, when the defendant is relatively more patient that the plaintiff (i.e. $\delta_D < \tilde{\delta}_P(\delta_D)$) there will be litigation, cases will go to trial, and the law will evolve. There are, however, two important differences. First, if the plaintiff is sufficiently patient, then the expected harms from legal evolution may deter the plaintiff from bringing cases at all, even in cases where the firm had produced a risky output that would have been litigated under the static benchmark. We say there is *regulatory capture.* The perceived costs of litigation enable the firm to behave with a degree of impunity.

Second, when both the plaintiff and defendant are patient, it may be that both the plaintiff is litigation averse and that the defendant is strongly incentivized to cause the law to evolve.

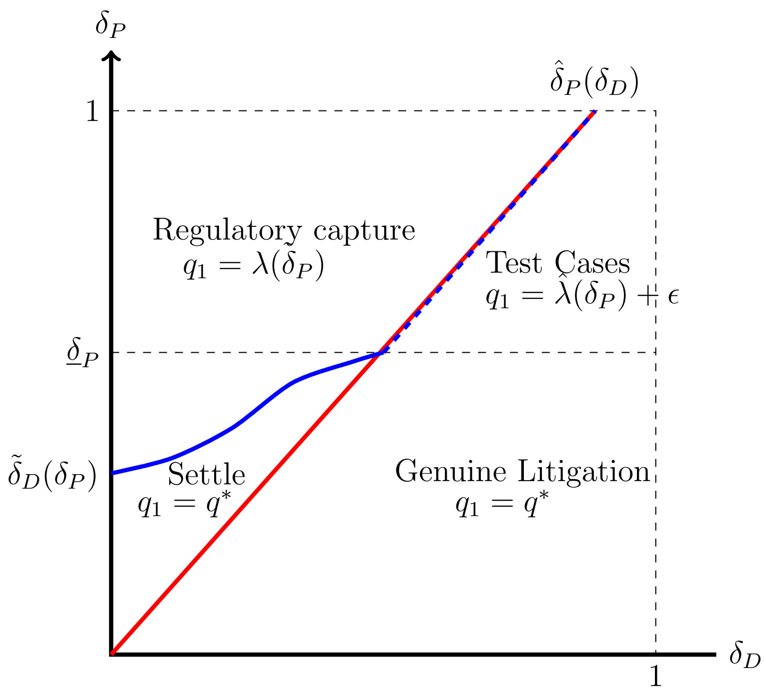

Figure 2: Firm's choice and litigation outcomes over the discount-parameter space, assuming that $u_1 - l_1 > 1 - u_1$ so that litigation arises along the equilibrium path in the static baseline.

In this case, the firm will bring 'test cases' by producing outputs just extreme enough as to force litigation along the equilibrium path. This insight mirrors the result in the baseline analysis where a sufficiently patient firm would produce increasingly risky outputs that made settlement less and less likely. Given the stark nature of learning in this model, it is sufficient for the firm to produce just beyond the threshold to ensure litigation along the equilibrium path.

6 Conclusion

We present an analytical model of the common law to study how plaintiff's and defendants can affect its evolution. Plaintiffs can prevent the law from evolving through settlement, and defendants can encourage the law to evolve in their favor, by generating test cases. These incentives create both static and dynamic inefficiencies (as noted earlier) so that the common law's evolution towards more efficient legal rules is not always guaranteed.

Plaintiffs wish to avoid learning, especially as they become more patient. Therefore, they desire to settle instead of going to court where the law will be clarified. When learning is complete (section 5.2), this aversion to learning can be so strong as to deter litigation altogether — even in cases where litigation would arise in the static baseline. When the plaintiff's learning costs are sufficiently high, a wedge open between the *de jure* and *de facto* safe outputs, in effect allowing the firm to produce higher outputs with impunity. We refer to this as a situation of regulatory capture.

By contrast, defendants have incentive to go to trial to clarify the law. Indeed, sufficiently patient defendants will choose quantities that invite litigation (or which make litigation more likely). In effect, they generate test cases which force the plaintiff's hand to go to trial in order to have the law clarified.

Whether cases go to trial or not depends critically on whether the plaintiffs or the defendants are more patient. Disputes that arise between relatively patient defendants and impatient plaintiffs go to trial. By contrast, disputes between impatient defendants and patient plaintiffs are resolved out of court. These results hold (broadly) regardless of whether the learning technology is complete or partial. These results are particularly apt in the context of a long-run oriented regulator/plaintiff interacting with short-run oriented firms/defendants. Our analysis formalizes intuitions offered previously by Galanter (1974) who notes that "repeat players to 'settle' cases where they expect unfavorable rule outcomes". Similarly, (Fiss, 1984) notes that "a settlement will thereby deprive a court of the occasion, and perhaps even the ability, to render an interpretation".

However, importantly this prior literature does not identify exactly, which parties (plaintiffs or defendants), or what benefits a repeat play may receive from settling cases. Our formalization, finds that it is primarily plaintiffs who will wish to settle to avoid clarifying the law, and that this effect will be especially strong when they are patient.

As an illustration of how settlement prevents the law from evolving consider the enforcement of Foreign Corrupt Practices Act (FCPA) violations. Around 92 % of FCPA cases violations settled pretrial with the SEC or DOJ.[9] And, indeed from a static efficiency perspective settlement is usually pareto superior to trial. However, dynamically such pre-trial settlements may not be efficient if they prevent the law from evolving. [10] Taken together, these results show that strategic concerns of long lived plaintiffs can affect the stream of cases that arise in court. This, in turn affects the type of cases that the court reviews, influencing the law's evolution and its efficacy.(Rubin (1977)).

Finally, this paper contributes to the broader literature on regulatory enforcement in two ways. First, instead of studying how systems and institutions affect actors which is typical in most of the regulatory literature, we study how actors affect the rules and the institutions such as the courts, while also being affected by those very institutions, which is far less studied. Second, much of the extant literature either focuses on either the role of courts or those of the regulator. Instead, in our framework enforcement can involve both the endogenous laws, the regulator, and the court. As our formal analysis shows, acknowledging these effects alters the impact of regulations. We believe that future work should consider these issues more deeply in other contexts.

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[9]Data is obtained from the Stanford Law School FCPA clearing house.

[10]Indeed, some legal scholars note that "FCPA enforcement is a legal desert, with guidance often drawn not from binding case law but from a whirl of enforcement patterns."

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Appendices

A Proofs

Proof of Lemma 1. First, notice that the firm will never produce $q < 1 - u$, since the marginal profit in this region is $1 - q > 0$. Similarly, the firm will never produce $q \geq 1 - l$, since the marginal profit in this region is $(1 - \frac{u+l}{2}) - q < 0$. Hence, there are two possibilities: either the firm produces the safe output $q = 1 - u$ or it produces some optimally chosen q in the ambiguous region $q \in (1 - u, 1 - l)$.

The marginal profit in the ambiguous region is:

$$\frac{\partial \pi}{\partial q} = 1 - q - \frac{u^2 - (1-q)^2}{2(u-l)} - \frac{q(1-q)}{u-l}$$

Solving this first order condition gives an optimal q of $q_A(l, u) = 1 - \frac{1-(u-l)+\sqrt{(1-(u-l))^2+3u^2}}{3}$. We can show that $q_A < q_E = 1 - \frac{u+l}{2} < 1 - l$, though there is no guarantee that $q_A > 1 - u$.

By the second order conditions, we know that $\frac{\partial \pi}{\partial q}$ is decreasing in q over the relevant range. Hence, to verify if $q_A > 1 - u$, it suffices to check the sign of $\frac{\partial \pi}{\partial q}$ at $q = 1 - u$. We have:

$$\frac{\partial \pi}{\partial q}|_{q=1-u} = u \left[1 - \frac{1-u}{u-l}\right]$$

Hence $\frac{\partial \pi}{\partial q} > 0$ and so $q^* = q_A > 1 - u$ if $1 - u < u - l$. If the latter condition does not hold, then π is decreasing over the entire ambiguous region, and so the safe output is optimal; $q^* = 1 - u$. ■

Lemma A. *Suppose $u_1 - l_1 < 2(1 - u_1)$. Then:*

- *If* $q_1 \in [\max\{\frac{1-l_1}{2}, 1-u_1\}, \min\{2(1-u_1), 1-l_1\}]$ *and there is learning, the second period beliefs* (l_2, u_2) *will imply a second period quantity at the corner:* $q^*(l_2, u_2) = 1 - u_2$.

- *If* $q_1 < \max\{\frac{1-l_1}{2}, 1-u_1\}$ *and there is learning, then the second period quantity will be at the corner if* q_1 *is found to be unreasonable, but will be interior if* q_1 *is found to be reasonable. I.e.* $q^*(1-q_1, u_1) = 1 - u_1$ *and* $q^*(l_1, 1-q_1) = q_A(l_1, 1-q_1) > q_1$.

Additionally, the following are true: (i) first period stage game efficient quantity $q_E = 1 - \frac{u_1+l_1}{2}$ *satisfies* $q_E \in [\max\{\frac{1-l_1}{2}, 1-u_1\}, \min\{2(1-u_1), 1-l_1\}]$*; (ii) if* $u_1 - l_1 < 1 - u_1$*, then* $q^*(l_1, u_1) = q_A(l_1, u_1) < \frac{1-l_1}{2}$*; and (iii)* $\max\{\frac{1-l_1}{2}, 1-u_1\} = 1-u_1$ *and* $\min\{2(1-u_1), 1-l_1\} = 1 - l_1$ *iff* $u_1 - l_1 < 1 - u_1$.

Proof of Lemma A. Suppose $q_1 \in [1-u, 1-l]$ is held to be acceptable. Then the new beliefs will be $(l_2, u_2) = (l_1, 1-q_1)$. Then by Lemma 1, the second period output $q^*(l_2, u_2)$ will be at the corner (i.e. $q^* = 1 - u_2 = q_1$) if $1 - q_1 - l_1 \leq q_1$, which implies that $q_1 \geq \frac{1-l_1}{2}$. Suppose, instead that q_1 is held to be unacceptable. Then the new beliefs will be $(l_2, u_2) = (1-q_1, u_1)$, and the second period output will be at the corner (i.e. $q^* = 1 - u_1$) if $u_1 - (1-q_1) \leq 1 - u_1$, which implies that $q_1 \leq 2(1-u_1)$.

Next, notice that if $u_1 - l_1 < 3(1-u_1)$, then $\frac{1-l_1}{2} < 2(1-u_1)$. If so, whenever the firm produces $q_1 \in [\max\{\frac{1-l_1}{2}, 1-u_1\}, \min\{2(1-u_1), 1-l_1\}]$, the second period output will be at a corner. Additionally, if $u_1 - l_1 < 2(1-u_1)$, as we assume, then it can be verified that this interval contains the stage game efficient output $q_E = 1 - \frac{u_1+l_1}{2}$. Additionally, Parameswaran (2018) shows that if $u_1 - l_1 > 1 - u_1$ so that $q^*(l_1, u_1) = q_A(l_1, u_1)$, then $q^* < \frac{1-l_1}{2}$. ■

Proof of Lemma 2. First, we show that $\mathcal{A}_P \geq 0$ and $\mathcal{A}_D \geq 0$. We omit time-subscripts wherever we can do so without confusion. First, consider $\mathcal{A}_D$. Recall that $\Pi(l, u) = \pi(q^*(l, u), l, u)$ and that $\pi(q, l, u) = q - \frac{1}{2}q^2 - I(q, l, u)$.

Then:

$$
\begin{aligned}
\mathcal{A}_D &= \frac{u-(1-q_1)}{u-l}\cdot\Pi(1-q_1,u)+\frac{(1-q_1)-l}{u-l}\cdot\Pi(l,1-q_1)-\Pi(l,u)\\
&\geq \frac{u-(1-q_1)}{u-l}\cdot\pi(q^*,1-q_1,u)+\frac{(1-q_1)-l}{u-l}\cdot\pi(q^*,l,1-q_1)-\pi(q^*,l,u)\\
&= I(q^*,l,u)-\frac{u-(1-q_1)}{u-l}\cdot I(q^*,1-q_1,u)+\frac{(1-q_1)-l}{u-l}\cdot I(q^*,l,1-q_1)\\
&= \left[\frac{u^2-(1-q^*)^2}{2(u-l)}-\frac{u-(1-q_1)}{u-l}\cdot\frac{u^2-(1-\min\{q_1,q^*\})^2}{2[u-(1-q_1)]}-\frac{(1-q_1)-l}{u-l}\cdot\frac{(1-q_1)^2-(1-\max\{q_1,q^*\})^2}{2((1-q_1)-l)}\right]q^*\\
&= \left[\frac{u^2-(1-q^*)^2}{2(u-l)}-\frac{u^2-(1-\min\{q_1,q^*\})^2}{2(u-l)}-\frac{(1-q_1)^2-(1-\max\{q_1,q^*\})^2}{2(u-l)}\right]q^*\\
&= 0
\end{aligned}
$$

where the second line makes use of the fact that $q^*(l,u)$ is generically not optimal for beliefs $(l,1-q_1)$ or $(1-q_1,u)$, and the fourth lines make use of the facts that: (i) $I(q^*,1-q,u)=\frac{u+(1-q)}{2}q^*$ if $q^*>q_1$ and $I(q^*,1-q,u)=\frac{u-(1-q^*)^2}{2[u-(1-q_1)]}q^*$ if $q^*<q_1$; and similarly (ii) $I(q^*,l,1-q_1)=\frac{u-(1-q^*)}{2[(1-q_1)-l]}q^*$ if $q^*>q_1$ and $I(q^*,1-q,u)=0$ if $q^*<q_1$.

Next, we must show that $\mathcal{A}_P\geq 0$. First suppose that $u-l\leq 1-u$. If so, we can easily verify that $\mathcal{A}_P(q_1)=\frac{(u-(1-q_1))(1-q-l)(1-q+l)}{2(u-l)}$. This is a cubic in q with roots at $1-u$, $1-l$ and $1+l$, and so it is non-negative over the interval $[1-u,1-l]$.

Suppose instead that $u-l>1-u$, so that $1-u<\frac{1-l}{2}<2(1-u)<1-l$. Over the interval $[1-u,\frac{1-l}{2}]$ we have:

$$
\mathcal{A}_P(q_1)=\frac{u^2-(1-q_1)^2}{2(u-l)}(1-u)+\frac{(1-q_1)-l}{u-l}\left[\frac{(1-q_1)+l}{2}q^*-I(q^*,l,1-q_1)\right]-J(q^*(l,u),l,u)
$$

where $q^* = q_A(l, 1-q_1)$. Notice that $\mathcal{A}_P(1-u) = 0$. Additionally:

$$\begin{aligned}
\mathcal{A}'_P(q_1) &= \frac{1-q_1}{u-l}\left[1-u-q^*\right] + \frac{I(q^*, l, 1-q_1)}{u-l} + \frac{1-q_1-l}{u-l} I_u(q^*, l, 1-q_1) + \\
&\quad + \left[\frac{(1-q_1)^2 - l^2}{2(u-l)} - \frac{1-q_1-l}{u-l}(1-q^*)\right]\frac{\partial q^*}{\partial q} \\
&= \frac{(1-q_1)(1-u)}{u-l} - \frac{1-q_1-l}{u-l}\left[q_E(l, 1-q_1) - q^*\right]\frac{\partial q^*}{\partial q}
\end{aligned}$$

where we make use of the fact that $I_u(q, l, u) = \frac{uq}{u-l} - \frac{I(q,l,u)}{u-l}$. By the implicit function theorem, we can show that $\frac{\partial q^*(l,1-q_1)}{\partial q_1} = \frac{q^*-q_1}{(1-q_1-l)+2-q^*} > 0$. Additionally, we know that $q_E - q^* < (1 - \frac{u+l}{2}) - (1-u) = \frac{u-l}{2} < \frac{1}{2}$. Hence we have:

$$\begin{aligned}
\mathcal{A}'_P(q_1) &= \frac{(1-q_1)}{u-l}\left[(1-u) - \frac{1-q_1-l}{1-q_1}\left[q_E(l, 1-q_1) - q^*\right]\frac{q^*-q_1}{(1-q_1-l)+2-q^*}\right] \\
&> \frac{(1-q_1)}{u-l}\left[(1-u) - (q_E(l, 1-q_1) - q^*)(q^*-q_1)\right] \\
&> \frac{(1-q_1)}{u-l}\left[(1-u) - \frac{1}{2}(u-l)\right] \\
&> 0
\end{aligned}$$

where the third line uses the fact that $q^* - q_1 < u - l$ (since $q^* < 1 - l$ and $q_1 > 1 - u$), and the fourth line uses the fact that $u - l < 2(1-u)$. Hence $A_P(q_1) > 0$ for all $q_1 \in (1-u, \frac{1-l}{2})$.

By an analogous argument, we can show that $\mathcal{A}'_P(q_1) < 0$ whenever $q_1 \in (2(1-u), 1-l)$ and that $\mathcal{A}_P(1-l) = 0$. Hence $A_P(q_1) > 0$ for all $q_1 \in (2(1-u), 1-l)$.

Finally, suppose $q_1 \in [\frac{1-l}{2}, 2(1-u)]$. Similar to the case of $u - l < 1 - u$, we can verify that $\mathcal{A}_P(q_1) = \frac{(u-(1-q_1))(1-q-l)(1-q+l)}{2(u-l)} - \Pi(l, u) + \frac{u+l}{2}(1-u)$. This again is a cubic which is strictly quasi-concave over the interval $[1-u, 1-l]$. Thus $\mathcal{A}_P(q_1) \geq 0$ for all $q_1 \in [\frac{1-l}{2}, 2(1-u)]$ if $\min\{\mathcal{A}_P(\frac{1-l}{2}), \mathcal{A}_P(2(1-u))\} \geq 0$. These properties were verified this above. ■

Proof of Lemma 3. The victim prefers to go to trial provided that: $I(q_1, l, u) > \delta_P \mathcal{A}_P(q_1, l, u)$, where $\mathcal{A}(q_1,l,u) = \frac{u-(1-q_1)}{u-l}J(1-q_1,u) + \frac{(1-q_1)-l}{u-l}J(l,1-q_1) - J(l,u)$. Since $\delta_P \in [0,1]$, it suffices to show that $\mathcal{A}(q_1,l,u) < I(q_1,l,u)$ for all $q_1 > 1-u$.

Recall, $J(l,u) = \frac{(1-q^*(l,u))^2-l^2}{2(u-l)}q^*(l,u)$ where $q^*(l,u) \geq 1-u$ is the firm's static optimal output. Since $q^*(l,u) \in [1-u,1-l)$, we must have $q^*(1-q_1,u) < q_1 \leq q^*(1-q_1,l)$ and $q^*(1-q_1,u) \leq q^*(l,u) \leq q^*(l,1-q_1)$. Since all quantities are positive, these implies that $(1-q^*(l,u))^2 \geq (1-q^*(l,1-q_1))^2$ and $u^2 \geq (1-q^*(1-q_1,u))^2$.

We have:

$$\begin{aligned}
\mathcal{A}_P(q_1,l,u) &= \frac{u-(1-q_1)}{u-l}\cdot\frac{(1-q^*(1-q_1,u))^2-(1-q_1)^2}{2(u-(1-q_1))}\cdot q^*(1-q_1,u) + \frac{(1-q_1)-l}{u-l}\cdot\frac{(1-q^*(l,1-q_1))^2-l^2}{2((1-q_1)-l)}\cdot q^*(l,1-
\\
&\quad - \frac{(1-q^*(l,u))^2-l^2}{2(u-l)}\cdot q^*(l,u)\\
&= \frac{(1-q^*(1-q_1,u))^2-(1-q_1)^2+(1-q^*(l,1-q_1))^2-(1-q^*(l,u))^2}{2(u-l)}\\
&< \frac{u^2-(1-q_1)^2}{2(u-l)}\\
&= I(q_1,l,u)
\end{aligned}$$

■

Lemma B. *$\mathcal{A}_D(q_1)$ is concave in q_1 for $q_1 \in [1-u,1-l]$.*

Proof of Lemma B. The proof is in two parts. First, we show that $\mathcal{A}_D$ is concave in the regions $q_1 \in [\max\{\frac{1-l}{2},1-u\},\min\{2(1-u),1-l\}]$, $q_1 \in [1-u,\max\{\frac{1-l}{2},1-u\})$ and $q_1 \in (\min\{2(1-u),1-l\},1-l]$. (Recall that if $u_1-l_1 < 1-u_1$, then the second and third regions are empty.

Take the first region. Note that pre-penalty profits are $q-\frac{1}{2}q^2 = \frac{1}{2}-\frac{1}{2}(1-q)^2$. Then by construction:

$$\mathcal{A}_D(q_1,l_1,u_1) = \frac{u_1-(1-q_1)}{u_1-l_1}\left[\frac{1}{2}-\frac{1}{2}u_1^2\right] + \frac{(1-q_1)-l_1}{u_1-l_1}\left[\frac{1}{2}-\frac{1}{2}(1-q_1)^2\right] - \Pi(l_1,u_1)$$

Hence:

$$\mathcal{A}_D'(q_1) = -\frac{u_1^2 - (1-q_1)^2}{2(u_1 - l_1)} + \frac{(1-q_1) - l_1}{u_1 - l_1}(1-q_1) = \frac{3(1-q_1)^2 - 2l_1(1-q_1) - u_1^2}{2(u_1 - l_1)}$$

and so:

$$\mathcal{A}_D''(q_1) = \frac{l_1 - 3(1-q_1)}{u_1 - l_1}$$

Notice that for any $q_1 \in [\max\{\frac{1-l}{2}, 1-u\}, \min\{2(1-u), 1-l\}]$, $1-q_1 \geq l_1$, and so $\mathcal{A}_D''(q_1) < 0$. Hence $\mathcal{A}_D$ is concave in this region.

Next, take the second region $[1-u, \max\{1-u, \frac{1-l}{2}\})$, and suppose that $u_1 - l_1 > 1 - u_1$ so that this region is non-empty. We have:

$$\mathcal{A}_D(q_1, l_1, u_1) = \frac{u_1 - (1-q_1)}{u_1 - l_1}\left[\frac{1}{2} - \frac{1}{2}u_1^2\right] + \frac{(1-q_1) - l_1}{u_1 - l_1}\left[\frac{1}{2} - \frac{1}{2}(1-q^*)^2 - I(q^*, l_1, 1-q_1)\right] - \Pi(l_1, u_1)$$

where $q^* = q_A(l_1, 1-q_1)$. Now, by the envelope theorem, and using the fact that $I_u(q^*(l,u), l, u) = \frac{uq^*}{u-l} - \frac{I(q^*,l,u)}{u-l}$, we have:

$$\begin{aligned}\mathcal{A}_D'(q_1) &= \frac{\frac{1}{2}(1-q^*)^2 - \frac{1}{2}u_1^2 + I(q^*, l_1, 1-q_1)}{u_1 - l_1} - \frac{1-q_1-l_1}{u_1-l_1} \cdot \left[\frac{I(q^*, l_1, 1-q_1)}{1-q_1-l_1} - \frac{(1-q_1)q^*}{1-q_1-l_1}\right] \\ &= \frac{(1-q_1)q^* - \frac{1}{2}[u_1^2 - (1-q^*)^2]}{u_1 - l_1}\end{aligned}$$

Then, noting (as we did in the proof of Lemma 2) that $\frac{\partial q^*(l_1, 1-q_1)}{\partial q_1} > 0$, we have:

$$\mathcal{A}_D''(q_1) = -\frac{q^*}{u_1 - l_1} - \frac{q^* - q_1}{u_1 - l_1} \cdot \frac{\partial q^*}{\partial q_1} < 0$$

which makes use of the fact that $q^*(l_1, 1-q_1) > q_1$. Hence $A_D(q_1)$ is concave in this region.

An analogous argument shows that the third region is concave as well.

Second, we show that $\mathcal{A}_D$ is smooth at the boundaries between the regions when $u_1 - l_1 >$

$1-u_1$. This guarantees that it is globally concave over $q_1 \in [1-u, 1-l]$. To see this, consider the left and right derivatives of $\mathcal{A}_D$ at $q_1 = \frac{1-l_1}{2}$. Recall that, by construction, when $q_1 = \frac{1-l_1}{2}$, then $q_A(l_1, 1-q_1) = q_1 = \frac{1-l_1}{2}$ We have:

$$\begin{aligned}
\lim_{q_1 \uparrow \frac{1-l_1}{2}} \mathcal{A}'_D(q_1) &= \frac{\left[\left(1-\frac{1-l_1}{2}\right)^2 - u_1^2\right] + 2\left(1-\frac{1-l_1}{2}\right)\frac{1-l_1}{2}}{2(u_1-l_1)} \\
&= \frac{\left[\left(\frac{1+l_1}{2}\right)^2 - u_1^2\right] + 2\frac{1-l_1}{2}\left(\frac{1+l_1}{2}\right)}{2(u_1-l_1)}
\end{aligned}$$

By contrast, we have:

$$\begin{aligned}
\lim_{q_1 \downarrow \frac{1-l_1}{2}} \mathcal{A}'_D(q_1) &= \frac{3\left(\frac{1+l_1}{2}\right)^2 - 2l_1\left(\frac{1+l_1}{2}\right) - u_1^2}{2(u_1-l_1)} \\
&= \frac{\left[\left(\frac{1+l_1}{2}\right)^2 - u_1^2\right] + 2\frac{1-l_1}{2}\left(\frac{1+l_1}{2}\right)}{2(u_1-l_1)} \\
&= \lim_{q_1 \uparrow \frac{1-l_1}{2}} \mathcal{A}'_D(q_1)
\end{aligned}$$

A similar approach verifies that there is a convex kink at $q_1 = 2(1-u)$, and that $\mathcal{A}_D$ is decreasing at this boundary. ■

Proof of Lemma 4. Suppose that $q_1 \in [\max\{\frac{1-l}{2}, 1-u\}, \min\{2(1-u), 1-l\}]$. We showed in the proof of Lemma B that:

$$\mathcal{A}'_D(q_1) = -\frac{u_1^2-(1-q_1)^2}{2(u_1-l_1)} + \frac{(1-q_1)-l_1}{u_1-l_1}(1-q_1) = \frac{3(1-q_1)^2 - 2l_1(1-q_1) - u_1^2}{2(u_1-l_1)}$$

Take the case of $q_1 = q_E$. By Lemma A we know that $q_E \in [\max\{\frac{1-l_1}{2}, 1-u_1\}, \min\{2(1-u_1), 1-l_1\}]$. Then, by direct substitution we have $\mathcal{A}'_D(q_E) = -\frac{5u_1+3l_1}{8} < 0$.

Next, take the case of $q_1 = q^*(l_1, u_1)$. If $u_1 - l_1 < 1 - u_1$, then by Lemmas 1 and A we know that $q^*(l_1,u_1) = 1-u_1 \in [\max\{\frac{1-l_1}{2}, 1-u_1\}, \min\{2(1-u_1), 1-l_1\}]$. Hence, again by direct substitution, $\mathcal{A}_D'(1-u_1) = u_1 > 0$.

If instead $u_1 - l_1 > 1-u_1$, then $q^*(l_1,u_1) = q_A(l_1,u_1) < \frac{1-l}{2}$. For $q_1 < \frac{1-l}{2}$, recall that:

$$\begin{aligned}
\mathcal{A}_D'(q_1) &= \frac{1-u_1^2}{2(u_1-l_1)} - \frac{\Pi(l_1, 1-q_1)}{u_1-l_1} - \frac{1-q_1-l_1}{u_1-l_1}\Pi_u(l_1,1-q_1) \\
&= \frac{1-u_1^2}{2(u_1-l_1)} - \left[\frac{1-(1-q_A(l_1,1-q_1))^2}{2(u_1-l_1)} - \frac{(1-q_1)^2-(1-q_A(l_1,1-q_1))^2}{2(1-q_1-l_1)(u_1-l_1)}q_A(l_1,1-q_1)\right] \\
&\quad - \frac{q_A(l_1,u_1)}{u_1-l_1}\left[\frac{(1-q_1)^2-(1-q_A(l_1,1-q_1))^2}{2(1-q_1-l_1)} - (1-q_1)\right] \\
&= -\frac{u_1^2-(1-q_A(l_1,1-q_1))^2}{2(u_1-l_1)} + \frac{(1-q_1)q_A(l_1,1-q_1)}{u_1-l_1}
\end{aligned}$$

Now, using the result from the proof of Lemma B:

$$\begin{aligned}
\mathcal{A}_D'\left(\frac{1-l_1}{2}\right) &= \frac{2\frac{1-l_1}{2}\left(\frac{1+l_1}{2}\right) + (\frac{1+l_1}{2})^2 - u_1^2}{2(u_1-l_1)} \\
&= \frac{1-u_1^2-\left(\frac{1-l_1}{2}\right)^2}{2(u_1-l_1)} \\
&> \frac{1-\left(\frac{2+l_1}{3}\right)^2-\left(\frac{1-l_1}{2}\right)^2}{2(u_1-l_1)} \\
&= \frac{1-\frac{25-2l_1+13l_1^2}{36}}{2(u_1-l_1)} \\
&> 0
\end{aligned}$$

where the first inequality uses the fact that $u_1 - l_1 < 2(1-u_1)$ (and hence $u_1 < \frac{2+l_1}{3}$), and the second inequality using the fact that $25 - 2l_1 + 13l_1^2 < 25 + 11l_1 < 36$ for $l_1 \in (0,1)$ (since $l_1^2 < l_1$). Hence $\mathcal{A}_D'\left(\frac{1-l_1}{2}\right) > 0$. Finally, since $\mathcal{A}_D''(q_1) < 0$, then $\mathcal{A}_D'(q_1) > 0$ for all $q_1 \in (1-u, \frac{1-l}{2})$ which includes the case of $q_1 = q_A(l_1,u_1)$. ■

Proof of Proposition 1. The firm chooses q_1 to maximize $\pi(q_1,l_1,u_1) + \delta_D\mathcal{A}_D(q_1,l_1,u_1)$, taking (l_1,u_1) as given. Since both $\pi(q_1)$ and $\mathcal{A}_D(q_1)$ are concave, the optimal choice is characterized by the first order condition. Denote $\phi(q_1) = \pi(q_1) + \delta_D \mathcal{A}_D(q_1)$. We have $\phi'(\hat{q}_1) \leq 0$ (with equality unless there is a corner solution at $q = 1 - u$).

First suppose $u_1 - l_1 < 1 - u_1$. We showed previously that $\mathcal{A}_D'(1 - u_1) = u_1 > 0$ and that $\pi'(1 - u) = u\left[1 - \frac{1-u}{u-l}\right] < 0$. Hence $\phi'(1 - u) \leq 0$ provided that $\delta_D \leq \frac{1-u}{u-l} - 1 = \overline{\delta}_D > 0$, and so $\hat{q}_1(\delta_D) = 1 - u$ if $\delta_D \leq \overline{\delta}_D$. If $\delta_D > \overline{\delta}_D$, then $\hat{q}_1 > 1 - u$ satisfies the first order condition $\phi'(\hat{q}_1) = 0$.

Now, suppose $u_1 - l_1 > 1 - u_1$. Since $\pi'(q_A) = 0$ and $\mathcal{A}_D'(q_A) > 0$, then $\phi'(q_A) > 0$ (unless $\delta_D = 0$). Similarly, since $\pi' < 0$ and $\mathcal{A}_D' < 0$ for $q \geq q_E$, then $\phi'(q_E) < 0$. Hence, by the intermediate value theorem, there exists $\hat{q}_1 \in (q_A, q_E)$ s.t. $\phi'(\hat{q}_1) = 0$. [Note briefly that $\hat{q}_1 \geq \frac{1-l_1}{2}$ provided that $\pi'\left(\frac{1-l_1}{2}\right) + \delta_D \mathcal{A}'\left(\frac{1-l_1}{2}\right) \geq 0$, which will be true if $\delta_D \geq -\frac{\pi'\left(\frac{1-l_1}{2}\right)}{\mathcal{A}_D'\left(\frac{1-l_1}{2}\right)} = \tilde{\delta}_D$.]

In both cases, since $\pi'' < 0$ and $\mathcal{A}_D'' < 0$, the implicit function theorem implies that:

$$\frac{\partial \hat{q}_1}{\partial \delta_D} = -\frac{\mathcal{A}_D'(\hat{q}_1)}{\phi''(\hat{q}_1)} > 0$$

■

Proof of Propositions 2.A and 2.B. By Proposition 1, we know that $\hat{q}_1(\delta_D) = 1 - u_1$ if both $u_1 - l_1 < 1 - u_1$ and $\delta_D \leq \overline{\delta}_D = \frac{1-u}{u-l} - 1$, and that otherwise $\hat{q}_1(\delta_D) \in (1 - u_1, q_E)$. By Lemma 3, we know that litigation is individually rational for the plaintiff provided that $q_1 > 1 - u_1$ (i.e. provided $\delta_D > \overline{\delta}_D$). Hence, there will be litigation along the equilibrium path unless $u_1 - l_1 \leq 1 - u_1$ and $\delta_D \leq \overline{\delta}_D$.

Next, suppose that litigation is individually rational for the plaintiff. By the settlement condition, the parties will settle if $\delta_P \geq \tilde{\delta}_P(\delta_D) = \frac{\mathcal{A}_D(\hat{q}_1(\delta_D))}{\mathcal{A}_P(\hat{q}_1(\delta_D))}\delta_D$.

Now, take the particular case in Lemma A of $u_1 - l_1 < 1 - u_1$ and $\delta_D > \overline{\delta}_D$. In this case, we know that $q^*(l_1, 1 - q_1) = q_1$ and $q^*(1 - q_1, u_1) = 1 - u_1$. Using these facts, we can verify that:

$$\mathcal{A}_D(q_1) = \frac{(u_1 + 1 - q_1)(u_1 - (1 - q_1)(1 - q_1 - l_1)}{2(u_1 - l_1)}$$
$$\mathcal{A}_P(q_1) = \frac{(l_1 + 1 - q_1)(u_1 - (1 - q_1)(1 - q_1 - l_1)}{2(u_1 - l_1)}$$

Hence, the settlement condition becomes:

$$\delta_P \geq \frac{u_1 + 1 - \hat{q}_1(\delta_D)}{l_1 + 1 - \hat{q}_1(\delta_D)}\delta_D = \tilde{\delta}_P(\delta_D)$$

Using the facts that $\frac{u_1+1-q_1}{l_1+1-q_1}$ is strictly increasing in q_1 and $\hat{q}_1$ is strictly increasing in δ_D (for $\delta_D > \overline{\delta}_D$), it is straightforward to verify that $\tilde{\delta}_P$ is strictly increasing in δ_D. ■

Proof of Lemma 5. Recall that the plaintiff will litigate if $I(q_1, l_1, u_1) > \delta_P \mathcal{B}_P(l_1, u_1)$. The left-hand side is continuous and strictly increasing in q_1, whilst the right-hand side is constant in q_1. Hence, there exists some $\hat{\lambda}$ such that the IR condition is satisfied whenever $q_1 > \hat{\lambda}(\delta_P, l_1, u_1)$. Since $I(1-u, l, u) = 0$, $\hat{\lambda}(0, l, u) = 1 - u$.

Now, the *de facto* safe output is implicitly defined by $I(\hat{\lambda}(\delta_P, l_1, u_1), l_1, u_1) = \delta_P \mathcal{B}_P(l_1, u_1)$. If $\mathcal{B}_P(l_1, u_1) > 0$, then the right-hand side is strictly increasing in δ_P and so it must be that $\hat{\lambda}$ is strictly increasing in δ_P as well. Moreover, if $I(q^*(l, u), l, u) < \mathcal{B}_P(l, u)$, then it must be that $\hat{\lambda}(1, l_1, u_1) > q^*(l_1, u_1)$. Thus it suffices to show that $\mathcal{B}_P(l_1, u_1) > I(q^*(l_1, u_1), l_1, u_1) \geq 0$, which by the definition of $\mathcal{B}_P$ is equivalent to showing that $E[\theta(1-\theta)] - E[\theta]q^*(l_1, u_1) > 0$.

There are two cases to consider. First, suppose $u_1 - l_1 < 1 - u_1$, so that $q^*(l_1, u_1) = 1 - u_1$.

Then:

$$\begin{aligned} E[\theta(1-\theta)] &= \int_{l_1}^{u_1} \theta(1-\theta) \cdot \frac{1}{u-l} d\theta \\ &> \int_{l_1}^{u_1} \theta(1-u_1) \cdot \frac{1}{u_1 - l_1} d\theta \\ &= q^*(l_1, u_1) E[\theta] \end{aligned}$$

as required.

Suppose instead $u_1 - l_1 > 1 - u_1$ so that $q^*(l_1, u_1) = q_A(l_1, u_1)$. Denote $x(l_1, u_1) = E[\theta(1-\theta)] - q_A(l_1, u_1)E[\theta]$. First, consider the special case of $l_1 = 0$. Then $x(0, u_1) = \frac{u_1}{6}(1 - 3u_1 + \sqrt{(1-u_1)^2 + 3u_1^2})$. With a little algebra, we can verify that $x(0, u_1) > 0$ whenever $u_1 - l_1 < 2(1 - u_1)$. Next, for any u_1, note that:

$$\frac{\partial x}{\partial l_1} = \frac{[1 + 3l_1 + 2l_1^2 - u_1 - 2l_1 u_1 + 3u_1^2]}{6\sqrt{(1 + l_1 - u_1)^2 + 3u_1^2}} + 1 - \frac{l_1 + u_1}{3}.$$

Clearly, the last two terms of the previous expression are positive (together). Now consider the first term. Again, the denominator is positive. The numerator, $1 + 3l_1 + 2l_1^2 - u_1 - 2l_1u_1 + 3u^2$, is increasing in l_1 (its derivative with respect to l_1 is $3 + 6l_1 - 2u_1 > 0$) and it is strictly positive at $l_1 = 0$. Thus, for any value of l, $\frac{\partial x}{\partial l_1} > 0$. This completes the proof. ■

Proof of Proposition 3. Fix some pair (δ_P, δ_D). Recall that litigation is only rational for the plaintiff if $q_1 > \hat{\lambda}(\delta_P)$. Furthermore, when litigation arises, the parties will settle if $\delta_P \leq \tilde{\delta}_P(\delta_D)$, and will go to trial otherwise.

Now, since the plaintiff makes a take-it-or-leave-it settlement offer, we know that the defendant will be held to their non-settlment utility in equilibrium (whether settlement occurs or not). Hence, the defendant's problem is to choose q_1 to maximize:

$$\begin{cases} q_1 - \frac{1}{2}q_1^2 + \delta_D \Pi(l_1, u_1) & \text{if } q_1 \leq \hat{\lambda}(\delta_P) \\ q_1 - \frac{1}{2}q_1^2 - I(q_1, l_1, u_1) + \delta_D[\Pi(l_1, u_1) + \mathcal{B}_D(l_1, u_1)] & \text{if } q_1 > \hat{\lambda}(\delta_P) \end{cases}$$

Clearly the firm will never choose $q_1 < \hat{\lambda}(\delta_P) \leq 1 - l_1$. Notice that the choice of q_1 does not affect expected second period profits at the margin — it merely determines whether litigation will arise or not.

The results will depend on whether the safe output is larger than the stage-optimal risky output or not. Since $\hat{\lambda}(\delta_P)$ is increasing in δ_P and $\hat{\lambda}(0) = 1 - u$, then there exists $\underline{\delta}_P$ s.t. $\hat{\lambda}(\delta_P) > q^*(l_1, u_1)$ provided that $\delta_P > \underline{\delta}_P$. (If $q^*(l_1, u_1) = 1 - u_1$, then $\underline{\delta}_P = 0$.)

There are two cases to consider. First, suppose $\delta_P > \underline{\delta}_P$ — i.e. $\hat{\lambda}(\delta_P) \geq q^*(l_1, u_1)$. Then, the firm has no first period incentive to produce $q_1 > \hat{\lambda}(\delta_P)$, and in fact, first period utility is decreasing in q_1 beyond $\hat{\lambda}(\delta_P)$. Additionally, though it may have a second period incentive, any $q_1 > \hat{\lambda}(\delta_P)$ would be just as good. Thus, the firm will either produce the *de facto* safe output $\hat{\lambda}(\delta_P)$ (which precludes litigation) or slightly more $\hat{\lambda}(\delta_P) + \varepsilon$ (which induces litigation). The former is preferred if: $\delta_D \leq \frac{I(\hat{\lambda}(\delta_P), l_1, u_1)}{\mathcal{B}_D(l_1, u_1)} = \frac{\mathcal{B}_P(l_1, u_1)}{\mathcal{B}_D(l_1, u_1)}\delta_P$, where we use the fact that $I(\hat{\lambda}(\delta_P), l_1, u_1) = \delta_P \mathcal{B}_P(l_1, u_1)$. But this is precisely the settlement condition.

Next, suppose $\delta_P \leq \underline{\delta}_P$ — i.e. $\hat{\lambda}(\delta_P) < q^*(l_1, u_1)$. Then the firm's must choose between the safe output $\hat{\lambda}(\delta_P)$ and the stage game optimal risky output $q^*(l_1, u_1) = q_A(l_1, u_1)$. The firm will make the safe choice provided that:

$$\hat{\lambda}(\delta_P) - \frac{1}{2}\hat{\lambda}^2(\delta_P) \geq \Pi(l_1, u_1) + \delta_D \mathcal{B}_D(l_1, u_1)$$

$$\hat{\lambda}(\delta_P) - \frac{1}{2}\hat{\lambda}^2(\delta_P) - I(\hat{\lambda}(\delta_P), l_1, u_1) \geq \Pi(l_1, u_1) + \delta_D \mathcal{B}_D(l_1, u_1) - \delta_P \mathcal{B}_P(l_1, u_1)$$

$$\delta_D \leq \frac{\mathcal{B}_P(l_1, u_1)}{\mathcal{B}_D(l_1, u_1)}\delta_P - \frac{\pi(q^*(l_1, u_1), l_1, u_1) - \pi(\hat{\lambda}(\delta_P), l_1, u_1)}{\mathcal{B}_D(l_1, u_1)}$$

Notice that $\pi(q^*(l_1,u_1)) > \pi(\hat{\lambda}(\delta_P))$ by construction, and that $\pi(q^*(l_1,u_1)) - \pi(\hat{\lambda}(\delta_P)) \to 0$ as $\delta_P \to \underline{\delta}_P$ Define:

$$\tilde{\delta}_D(\delta_P) = \frac{\mathcal{B}_P(l_1,u_1)}{\mathcal{B}_D(l_1,u_1)}\delta_P - \frac{\max\{0, \pi(q^*(l_1,u_1),l_1,u_1) - \pi(\hat{\lambda}(\delta_P),l_1,u_1)\}}{\mathcal{B}_D(l_1,u_1)}$$

Thus, the firm will choose the safe output $\hat{\lambda}(\delta_P)$ if $\delta_D \leq \tilde{\delta}_D(\delta_P)$, and a risky output if $\delta_D > \tilde{\delta}_D(\delta_P)$. The risky output will be $q_A(l_1,u_1)$ if $\delta_P < \underline{\delta}_P$, and it will be $\hat{\lambda}(\delta_P) + \varepsilon$ if $\delta_P \geq \underline{\delta}_P$ ■